



K22P 3319

Reg. No. :

Name :

IV Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.)

Examination, April 2022

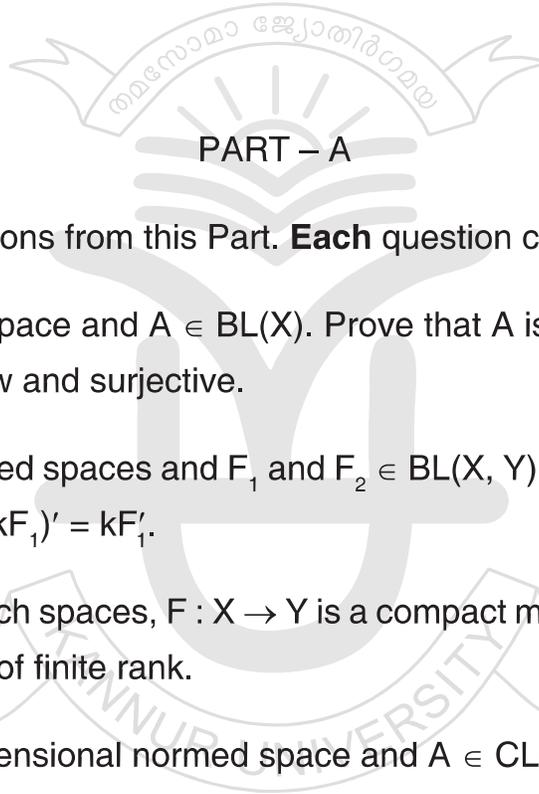
(2018 Admission Onwards)

MATHEMATICS

MAT4C15 : Operator Theory

Time : 3 Hours

Max. Marks : 80



Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Let X be a normed space and $A \in BL(X)$. Prove that A is invertible if and only if A is bounded below and surjective.
2. Let X and Y be normed spaces and F_1 and $F_2 \in BL(X, Y)$ and $k \in K$. Show that $(F_1 + F_2)' = F_1' + F_2'$, $(kF_1)' = kF_1'$.
3. Let X and Y be Banach spaces, $F : X \rightarrow Y$ is a compact map and $R(F)$ is closed in Y . Prove that F is of finite rank.
4. If X is an infinite dimensional normed space and $A \in CL(X)$. Prove that $0 \in \sigma_a(A)$.
5. Let H be a Hilbert space. If each (A_n) is self adjoint operator in $BL(H)$ and $\|A_n - A\| \rightarrow 0$, then prove that A is self adjoint.
6. Prove that the adjoint of Hilbert Schmidt operator on a separable Hilbert space is Hilbert Schmidt operator. **(4×4=16)**

P.T.O.



PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

UNIT – I

7. a) Let X be a normed space and $A \in BL(X)$ be of finite rank. Show that $\sigma_e(A) = \sigma_a(A) = \sigma(A)$.
- b) Let X be a Banach space over K and $A \in BL(X)$. Show that $\sigma(A)$ is a compact subset of K .
8. a) Let X be a normed space and X' is separable, prove that X is separable.
- b) Let $1 \leq p \leq \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Show that the dual of K^n with the norm $\| \cdot \|_p$ is linearly isomorphic to K^n with the norm $\| \cdot \|_q$.
9. a) Let X be a normed space and (x_n) be a sequence in X . Then prove that (x_n) is weak convergent in X if and only if
- (x_n) is a bounded sequence in X and
 - there is some $x \in X$ such that $x'(x_n) \rightarrow x'(x)$ for every x' in some subset of X' whose span is dense in X' .
- b) Let (x'_n) be a sequence in a normed space X' . if
- (x'_n) is bounded and
 - $(x'_n(x))$ is a Cauchy sequence in K for each x in a subset of X whose span is dense in X .
- Then, prove that (x'_n) is weak* convergent in X' . Is the converse true? Justify your answer.

UNIT – II

10. a) Let X be a reflexive normed space. Prove that every closed subspace of X is reflexive.
- b) Examine the reflexivity of $L^p([a, b])$, $1 \leq p \leq \infty$.
11. a) When a normed space X is said to be uniformly convex?
- b) Let X be a Banach space which is uniformly convex in some equivalent norm. Then prove that X is reflexive. Is the converse true? Justify your answer.



12. Let X be a normed space, Y be a Banach space and $F \in BL(X, Y)$, then prove that
- $CL(X, Y)$ is a closed subspace of $BL(X, Y)$.
 - $F \in CL(X, Y)$ if and only if $F' \in CL(Y', X')$.

UNIT – III

13. Let H be a Hilbert space and $A \in BL(H)$. Then prove the following,
- A is injective if and only if $R(A^*)$ is dense in H .
 - The closure of $R(A)$ equals $Z(A^*)^\perp$.
 - $R(A) = H$ if and only if A^* is bounded below.
14. Let H be a Hilbert space and $A \in BL(H)$.
- If A is normal, x_1 and x_2 are eigenvectors of A corresponding to distinct eigenvalues, then prove that $x_1 \perp x_2$.
 - Prove that every spectral value of A is an approximate eigenvalue of A .
 - Define the numerical range of A and show that it is bounded, but not closed.
15. Let A be compact operator on non-zero Hilbert space.
- Prove that non-zero approximate eigenvalue of A is an eigenvalue of A and the corresponding eigenspace is finite dimensional.
 - If A is self adjoint, then prove that $\|A\|$ or $-\|A\|$ is an eigenvalue of A .
- (4×16=64)**



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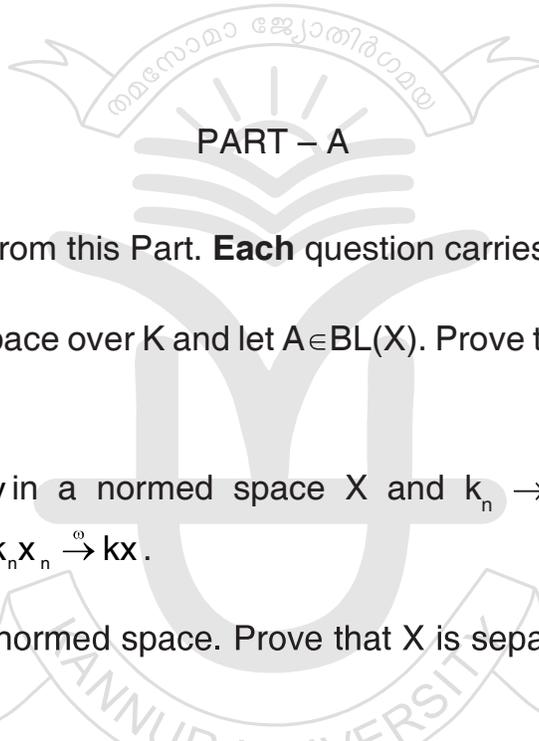
Reg. No. :

Name :

**IV Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT4C15 : Operator Theory**

Time : 3 Hours

Max. Marks : 80



Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Let X be a Banach space over K and let $A \in BL(X)$. Prove that $\sigma(A)$ is a compact subset of K .
2. If $x_n \xrightarrow{o} x$ and $y_n \xrightarrow{o} y$ in a normed space X and $k_n \rightarrow k$ in K , prove that $x_n + y_n \xrightarrow{o} x + y$ and $k_n x_n \xrightarrow{o} kx$.
3. Let X be a reflexive normed space. Prove that X is separable if and only if X' is separable.
4. Let X and Y be a normed spaces and $F : X \rightarrow Y$ be linear. Prove that F is a compact map if and only if for every bounded sequence (x_n) in X , $(F(x_n))$ has a subsequence which converges in Y .
5. Let H be a Hilbert space and $A \in BL(H)$. Prove that A is normal if and only if $\|A(x)\| = \|A^*(x)\|$ for all $x \in H$.
6. Let $A \in BL(H)$. If A is compact, prove that A^* is also compact.

P.T.O.



PART – B

Answer **four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. Let X be a nonzero Banach space over \mathbb{C} and $A \in BL(X)$. Prove that
- $\sigma(A)$ is non empty.
 - $r_\sigma(A) = \inf_{n=1,2,\dots} \|A^n\|^{\frac{1}{n}} = \lim_{n \rightarrow \infty} \|A^n\|^{\frac{1}{n}}$.
8. a) Let X be a normed space and $A \in BL(X)$ be of finite rank. Prove that $\sigma_e(A) = \sigma_a(A) = \sigma(A)$.
- b) Let X, Y and Z be normed spaces. Let $F \in BL(X, Y)$ and $G \in BL(Y, Z)$. Prove that
- $(GF)' = F'G'$
 - $\|F'\| = \|F\| = \|F''\|$ and
 - $F'' J_X = J_Y F$.
9. a) Let X be a normed space. If X' is separable, prove that X is separable.
- b) Prove that $x_n \xrightarrow{\omega} x$ in l^1 if and only if $x_n \rightarrow x$ in l^1 .

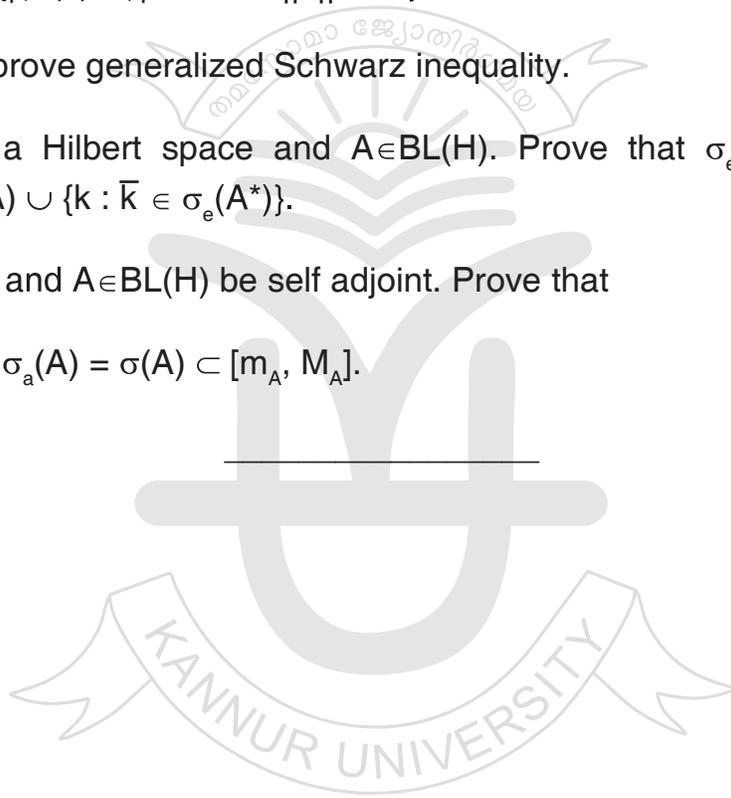
Unit – II

10. Let X be a normed space. Prove that X is reflexive if and only if every bounded sequence in X has a weak convergent subsequence.
11. a) Let X be a uniformly convex normed space and (x_n) be a sequence in X such that $\|x_n\| \rightarrow 1$ and $\|x_n + x_m\| \rightarrow 2$ as $m, n \rightarrow \infty$. Prove that (x_n) is a Cauchy sequence.
- b) Let X and Y be normed spaces and $F \in BL(X, Y)$. If $F \in CL(X, Y)$, prove that $F' \in CL(X, Y)$. Also show that the converse holds if Y is a Banach space.
12. Let X be a normed space and $A \in CL(X)$. Prove that $\dim Z(A' - kI) = \dim Z(A - kI) < \infty$ for $0 \neq k \in K$.



Unit – III

13. a) Let H be a Hilbert space and $A \in BL(H)$. Prove that there is a unique $B \in BL(H)$ such that for all $x, y \in H$, $\langle A(x), y \rangle = \langle x, B(y) \rangle$.
- b) Let H be a Hilbert space and $A \in BL(H)$. Prove that $R(A) = H$ if and only if A^* is bounded below.
14. a) Let H be a Hilbert space and $A \in BL(H)$. Let A be self adjoint. Prove that $\|A\| = \sup \{ |\langle A(x), x \rangle| : x \in H, \|x\| \leq 1 \}$.
- b) State and prove generalized Schwarz inequality.
15. a) Let H be a Hilbert space and $A \in BL(H)$. Prove that $\sigma_e(A) \subset \sigma_a(A)$ and $\sigma(A) = \sigma_a(A) \cup \{k : \bar{k} \in \sigma_e(A^*)\}$.
- b) Let $H \neq \{0\}$ and $A \in BL(H)$ be self adjoint. Prove that $\{m_A, M_A\} \subset \sigma_a(A) = \sigma(A) \subset [m_A, M_A]$.





K24P 0319

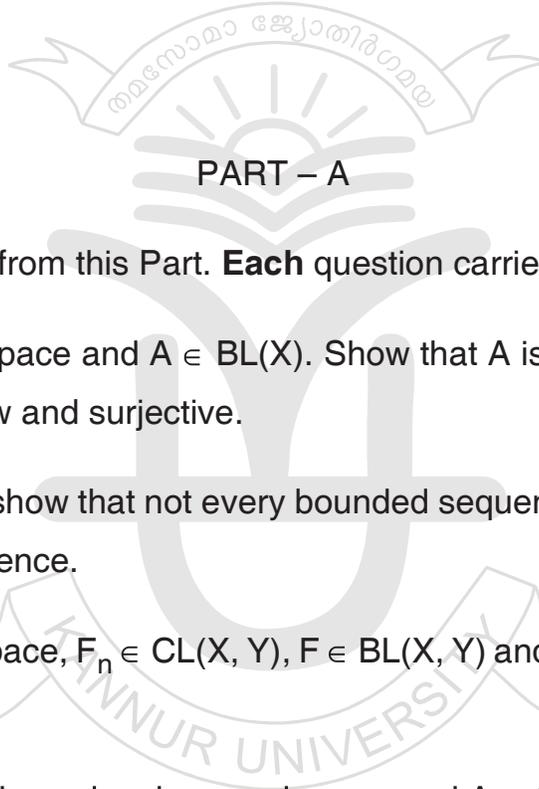
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**IV Semester M.Sc. Degree (CBSS – Reg./Supple.-(One Time Mercy
Chance)/Imp.) Examination, April 2024
(2017 Admission Onwards)
MATHEMATICS
MAT4C15 : Operator Theory**

Time : 3 Hours

Max. Marks : 80



Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Let X be a normed space and $A \in BL(X)$. Show that A is invertible if and only if A is bounded below and surjective.
2. Give an example to show that not every bounded sequence in X' has a weak* convergent subsequence.
3. Let Y be a Banach space, $F_n \in CL(X, Y)$, $F \in BL(X, Y)$ and $\|F_n - F\| \rightarrow 0$. Prove that $F \in CL(X, Y)$.
4. Let X be an infinite dimensional normed space and $A \in CL(X)$. Prove that $0 \in \sigma_a(A)$.
5. Let H be a Hilbert space. Consider $A, B \in BL(H)$, prove that $(A + B)^* = A^* + B^*$ and $(AB)^* = B^*A^*$.
6. Let H be a Hilbert space and $A \in BL(H)$. Prove that A is normal if and only if $\|A(x)\| = \|A^*(x)\|$ for all $x \in H$. **(4×4=16)**

P.T.O.



PART – B

Answer **four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. Let $X = l^p$ with the norm $\| \cdot \|_p$, $1 \leq p \leq \infty$. For $x = (x(1), x(2), \dots) \in X$, let $C(x) = (0, x(1), x(2), \dots)$. Find $\sigma(C)$, $\sigma_e(C)$ and $\sigma_a(C)$.
8. Prove that dual of l^1 is l^∞ .
9. a) Let X be a Banach space, $A \in BL(X)$ and $\|A\|^p < 1$ for some positive integer p . Show that $I - A$ is invertible and $(I - A)^{-1} = \sum_{n=0}^{\infty} A^n$.
- b) Show that $x_n \xrightarrow{w} x$ in l^1 if and only if $x_n \rightarrow x$ in l^1 .

Unit – II

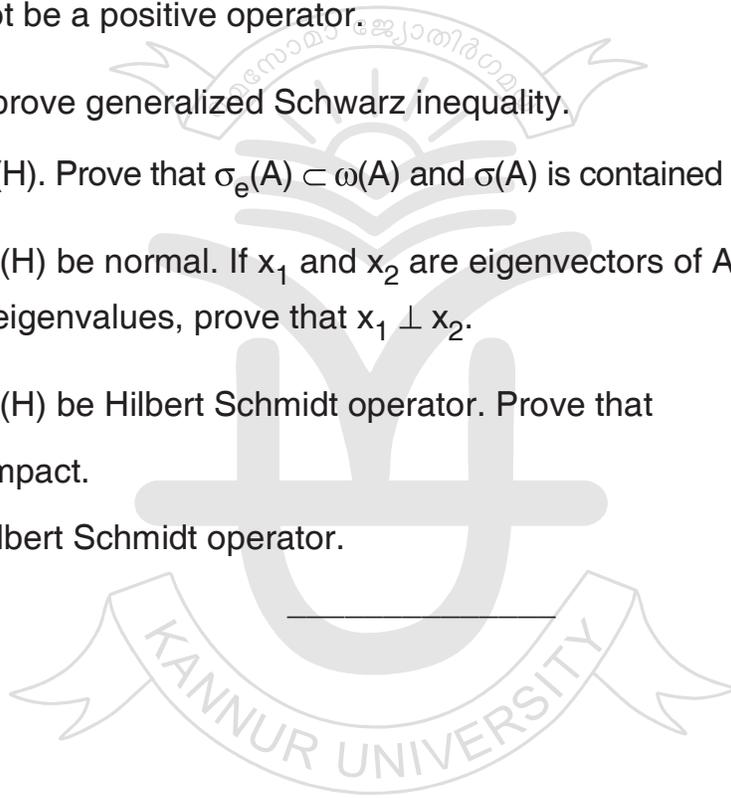
10. Let X be a reflexive normed space. Prove that
- X is Banach and it remains reflexive in any equivalent norm
 - X' is reflexive
 - Every closed subspace of X is reflexive
 - X is separable if and only if X' is separable.
11. a) Let X be a Banach space which is uniformly convex in some equivalent norm. Prove that X is reflexive.
- b) Let X and Y be normed spaces and $F \in BL(X, Y)$. If $F \in CL(X, Y)$, show that $F' \in CL(Y', X')$.
12. Let X be a normed space and $A \in CL(X)$. Prove that every nonzero spectral value of A is an eigenvalue of A .



Unit – III

13. a) Let H be a Hilbert space and $A \in BL(H)$. Prove that $R(A) = H$ if and only if A^* is bounded below.
- b) Let H be a Hilbert space and $A \in BL(H)$. Prove that A is unitary if and only if $\|A(x)\| = \|x\|$ for all $x \in H$ and A is surjective.
- c) Give examples of positive operators A and B such that composition operators AB may not be a positive operator.
14. a) State and prove generalized Schwarz inequality.
- b) Let $A \in BL(H)$. Prove that $\sigma_e(A) \subset \omega(A)$ and $\sigma(A)$ is contained in closure of $\omega(A)$.
15. a) Let $A \in BL(H)$ be normal. If x_1 and x_2 are eigenvectors of A corresponding to distinct eigenvalues, prove that $x_1 \perp x_2$.
- b) Let $A \in BL(H)$ be Hilbert Schmidt operator. Prove that
- i) A is compact.
 - ii) A^* is Hilbert Schmidt operator.

(4×16=64)





K24P 0320

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**IV Semester M.Sc. Degree (C.B.S.S. – Reg./Supple. – (One Time Mercy
Chance)/Imp.) Examination, April 2024
(2017 Admission Onwards)
MATHEMATICS
MAT4C16 : Differential Geometry**

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **four** questions from this part. **Each** question carries **4** marks. **(4×4=16)**

1. Sketch the gradient vector field of the function $f(x_1, x_2) = x_1^2 + x_2^2$.
2. Sketch the graph of the function $f(x_1, x_2) = x_1^2 - x_2^2$.
3. Prove that $X \dot{+} Y = \dot{X} + \dot{Y}$.
4. Define (i) Radius of Curvature (ii) Circle of Curvature, of a plane curve C.
5. Explain why unit speed curves are parametrized by arc length.
6. Find the length of the parametrized curve $\alpha(t) = (\sin t, \cos t, \sin t, \cos t)$ in $[0, 2\pi]$.

PART – B

Answer **four** questions from this part without omitting **any** Unit. **Each** question carries **16** marks. **(4×16=64)**

Unit – I

7. a) Find the integral curve through $(1, -1)$ of the vector field $X(x_1, x_2) = (x_1, x_2, -x_2, -\frac{1}{2}x_1)$.
- b) Show that the unit n sphere is an n surface in R^{n+1} .
- c) Sketch the typical level curves for $c = -1, 0, 1$ and graph of the function $f(x_1, x_2) = x_1 - x_2^2$.

P.T.O.



8. a) Prove the following : Let $S = f^{-1}(c)$ be an n – surface in \mathbb{R}^{n+1} , where $f : U \rightarrow \mathbb{R}$ is such that $\nabla f(q) \neq 0$ for all $q \in S$, and let X be a smooth vector field on U whose restriction to S is a tangent vector field on S . If $\alpha : I \rightarrow U$ is any integral curve of X such that $\alpha(t_0) \in S$ for some t_0 in I , then $\alpha(t) \in S$ for all $t \in I$.
- b) Find the maximum value and minimum value of the function $g(x_1, x_2) = ax_1^2 + bx_2^2 + 2x_1x_2$, $x_1, x_2 \in \mathbb{R}$ on the unit circle $x_1^2 + x_2^2 = 1$.
- c) Find the orientations on the cylinder $x_1^2 + x_3^2 = 1$ in \mathbb{R}^3 .
9. a) Find and sketch the gradient field of the function $f(x_1, x_2) = x_1^2 + x_2^2$.
- b) i) Verify that a surface of revolution is a 2 – surface.
ii) Sketch the surface of revolution obtained by rotating the curve $x_2 = 2$.
- c) Show that graph of any function $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a level set for some function $F : \mathbb{R}^{n+1} \rightarrow \mathbb{R}$.

Unit – II

10. a) Prove the following : Let S be an n surface in \mathbb{R}^3 and $\alpha : I \rightarrow S$ be a geodesic in S with $\dot{\alpha} \neq 0$. Then a vector field X tangent to S along α is parallel along α if and only if both $\|X\|$ and the angle between X and $\dot{\alpha}$ are constant along α .
- b) Compute the Weingarten map for the circular cylinder $x_2^2 + x_3^2 = a^2$ in \mathbb{R}^3 ($a \neq 0$).
11. a) Prove the following :
- $\nabla_v(X + Y) = \nabla_v(X) + \nabla_v(Y)$
 - $\nabla_v(fX) = (\nabla_v f) X(p) + f(p)(\nabla_v X)$
 - $\nabla_v(X \cdot Y) = (\nabla_v X) \cdot Y(p) + X(p) \cdot (\nabla_v Y)$
- b) With the usual notations, prove that $L_p(v) \cdot w = L_p(w) \cdot v$, $\forall v, w \in S_p$.
- c) With the usual notations, prove that the parallel transport $P_\alpha : S_p \rightarrow S_q$ along α is a vector space isomorphism which preserves dot product.
12. a) Find the curvature of the plane curve $C = f^{-1}(0)$ oriented by the outward normal where $f(x_1, x_2) = x_2^2 - x_1$.
- b) Show that i) $D_v(fX) = (\nabla_v f) X(p) + f(p)D_v X$
ii) $\nabla_v(X \cdot Y) = (D_v X) \cdot Y(p) + X(p) \cdot (D_v Y)$



Unit – III

13. a) Prove the following : Let η be the 1 – form on $\mathbb{R}^2 - \{0\}$ defined by
$$\eta = - \frac{x_2}{x_1^2 + x_2^2} + \frac{x_1}{x_1^2 + x_2^2}$$
. Then for $\alpha : [a, b] \rightarrow \mathbb{R}^2 - \{0\}$ be any closed piece wise smooth parametrized curve in $\mathbb{R}^2 - \{0\}$, $\int_{\alpha} \eta = 2\pi k$.
- b) Find the Gaussian curvature of the surface $x_1^2 + x_2^2 - x_3 = 0$ oriented by its outward normal.
14. a) Derive the formula for Gaussian curvature of an oriented n – surface in \mathbb{R}^{n+1} .
- b) Prove the following : Let S be an n – surface in \mathbb{R}^{n+1} and let $p \in S$. Then there exists an open set V about $p \in \mathbb{R}^{n+1}$ and a parametrized n surface $\phi : U \rightarrow \mathbb{R}^{n+1}$ such that ϕ is a one one map from U on to $S \cap V$.
15. a) Obtain a Torus as a parametrized surface in \mathbb{R}^3 .
- b) Prove the following : Let S be an n surface in \mathbb{R}^{n+1} and let $f : S \rightarrow \mathbb{R}^k$. Then f is smooth if and only if $f \circ \phi : U \rightarrow \mathbb{R}^k$ is smooth for each local parametrization $\phi : U \rightarrow S$.
- c) Let V be a finite dimensional vector space with dot product and let $L : V \rightarrow V$ be a self adjoint linear transformation on V . Prove that there exist an orthonormal basis for V consisting of eigenvectors of L .





K23P 0205

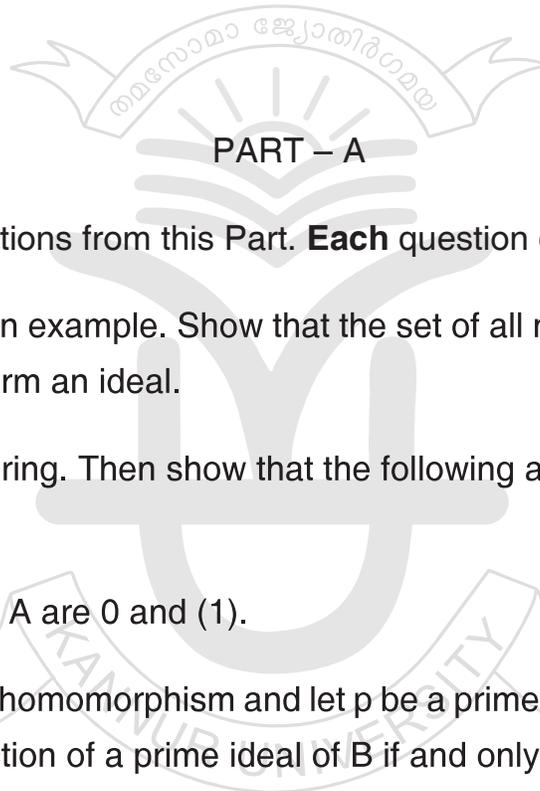
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IV Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.)
Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT 4E01 : Commutative Algebra

Time : 3 Hours

Max. Marks : 80



PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Define ideal. Give an example. Show that the set of all nilpotent elements of a commutative ring form an ideal.
2. Let A be a nonzero ring. Then show that the following are equivalent.
 - i) A is a field
 - ii) The only ideal in A are 0 and (1) .
3. Let $A \rightarrow B$ be a ring homomorphism and let p be a prime ideal of A . Then prove that p is the contraction of a prime ideal of B if and only if $p^{ec} = p^c$.
4. Let q be a prime ideal in a ring A . Show that $r(q)$ is the smallest prime ideal containing q .
5. If $x \in B$ is integral over A , show that $A[x]$ is finitely generated A – module.
6. Let $A \subset B$ be integral domains, B integral over A . Prove that B is a field if and only if A is field.

P.T.O.



PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Show that if $x \in \mathcal{R}$, the Jacobson radical if and only if $1 - xy$ is a unit in $A, \forall y \in A$.
- b) Show that the set \mathcal{R} of all nilpotent elements in a ring A is an maximal ideal, and $A \setminus \mathcal{R}$ has no nonzero nilpotent elements.
8. a) Let p_1, p_2, \dots, p_n be prime ideals and let α be an ideal contained in $\bigcup_{i=1}^n p_i$. Then show that $\alpha \subset p_i$, for some i .
- b) Let $\alpha_1, \alpha_2, \dots, \alpha_n$ be ideals and p be a prime ideal containing $\bigcap_{i=1}^n \alpha_i$, then prove that $p \supseteq \alpha_i$, for some i . If $p = \bigcap \alpha_i$, then prove that $p = \alpha_i$, for some i .
9. a) If $L \supset M \supset N$ are A -modules, then show that $(L/N)/(M/N) \cong L/M$.
- b) M_1 and M_2 are sub modules of M , then show that $(M_1 + M_2)/M_1 \cong M_2/(M_1 \cap M_2)$.

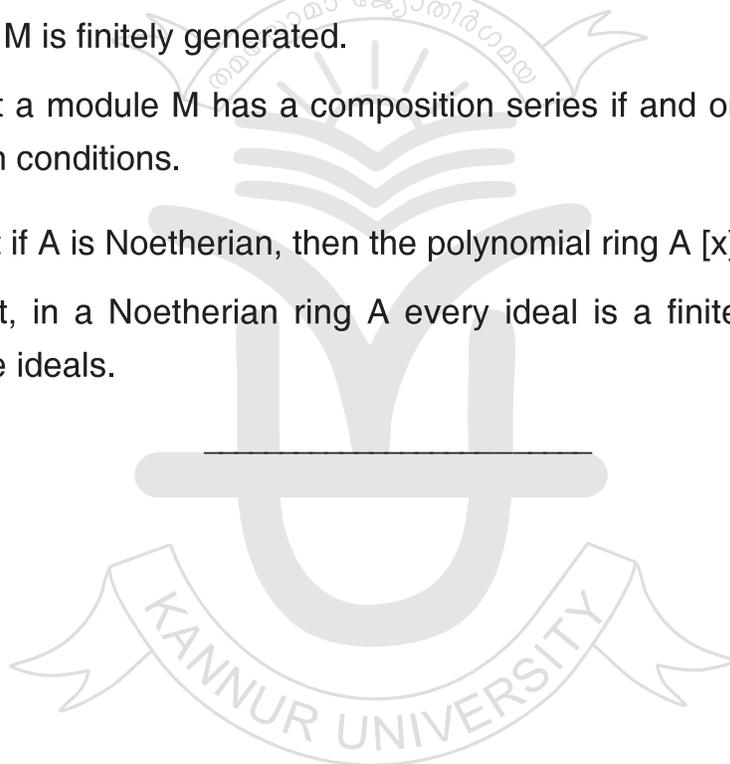
Unit – II

10. a) Let M be a A – module. Then prove that the following are equivalent.
- $M = 0$
 - $M_p = 0$ for all prime ideals p of A
 - $M_m = 0$ for all maximal ideals m of A .
- b) Let q be a prime ideal in a ring A , then show that $r(q)$ is the smallest prime ideal containing q .
11. a) State and prove first uniqueness theorem.
- b) Show that if A is absolutely flat, then every prime ideal is maximal.
12. Let S be a multiplicatively closed subset of A , and let q be a p -primary ideal. Then show that the followings
- If $S \cap p \neq \phi$, then $S^{-1}q = S^{-1}A$.
 - If $S \cap p = \phi$, then $S^{-1}q$ is $S^{-1}p$ - primary and its contraction in A is q .



Unit – III

13. a) Let $A \subset B$, be rings, B is integral over A , let q, q' be prime ideals of B such that $q \subset q'$ and $q^c \subset q'^c$ say, then show that $q = q'$.
- b) Let $A \subset B$ be rings, C the integral closure of A in B . Let S be a multiplicatively closed subset of A , then prove that $S^{-1}C$ is the integral closure of $S^{-1}A$ in $S^{-1}B$.
14. a) Show that the module M is a Noetherian A – module if and only if every sub module of M is finitely generated.
- b) Show that a module M has a composition series if and only if M satisfies both chain conditions.
15. a) Show that if A is Noetherian, then the polynomial ring $A[x]$ is also Noetherian.
- b) Show that, in a Noetherian ring A every ideal is a finite intersection of irreducible ideals.





K24P 0321

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**IV Semester M.Sc. Degree (C.B.S.S. – Reg./Supple.-(One Time Mercy
Chance)/Imp.) Examination, April 2024
(2017 Admission Onwards)**

MATHEMATICS

MAT 4E01 : Commutative Algebra

Time : 3 Hours

Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Define maximal ideal. If F is a field then show that only ideals are $\{0\}$, and F itself.
2. Let a be a nilpotent element of A . Prove that $1 + a$ is a unit of A . Deduce that the sum of a nilpotent element and a unit is a unit.
3. Let $g : A \rightarrow B$ be a ring homomorphism such that $g(s)$ is a unit in B for all $s \in S$, then show that there exists a unique ring homomorphism $h : S^{-1}A \rightarrow B$ such that $g = h \circ f$.
4. Show that if, $r(a)$ is maximal then a is primary. In particular, the powers of a maximal ideal m are m -primary.
5. If $A \subset B \subset C$ are rings and, if B is integral over A , and C is integral over B , then show that C is integral over A .
6. Show that, every finite abelian group satisfies both a.c.c. and d.c.c. **(4×4=16)**

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Let A be ring and $m \neq (1)$ is an ideal of A such that every $x \in A - m$ is a unit in A . Then prove that A is local ring and m its maximal ideal.
b) Let A be a ring and m a maximal ideal of A , such that every element $1 + x$, $x \in m$, is a unit in A , then show that A is a local ring.

P.T.O.



8. a) Show that M is finitely generated A -module if and only if M is isomorphic to a quotient of A^n , for some $n > 0$.
- b) Let M be a finitely generated A -module and α an ideal of A contained in the Jacobson radical R of A , then show that $\alpha M = M$ implies $M = 0$.
9. a) Let x_i , $1 \leq i \leq n$, be elements of M whose images in M/mM form a basis of this vector space. Show that the x_i generate M .
- b) Let $0 \rightarrow M_0 \rightarrow M_1 \rightarrow \dots \rightarrow M_n \rightarrow 0$ be an exact sequence of A -modules in which all the modules M_i and the kernels of all the homomorphism belong to C . Then for any additive function λ on C show that

$$\sum_{i=0}^n (-1)^i \lambda(M_i) = 0.$$

Unit – II

10. a) If N and P are sub modules of an A -module M , then prove the followings.
- $S^{-1}(N + P) = S^{-1}(N) + S^{-1}(P)$
 - $S^{-1}(N \cap P) = S^{-1}(N) \cap S^{-1}(P)$.
- b) Show that every ideal in $S^{-1}A$ is an extended ideal.
11. a) Let M be a finitely generated A -module, S a multiplicatively closed subset of A , show that $S^{-1}(\text{Ann}(M)) = \text{Ann}(S^{-1}(M))$.
- b) Let q be a prime ideal in a ring A , then show that $r(q)$ is the smallest prime ideal containing q .
12. a) State and prove first uniqueness theorem.
- b) Show that, the isolated primary components (i, e., the primary components q_i corresponding to minimal prime ideals p_i) are uniquely determined by α .

Unit – III

13. a) Show that the followings are equivalent.
- $x \in B$ is integral over A .
 - $A[x]$ is a finitely generated A -module.
 - $A[x]$ is contained in a subring C of B such that C is a finitely generated A -module.
 - There exists a faithful $A[x]$ -module M which is finitely generated as an A -module.
- b) Let $A \subset B$ be rings and let C be the integral closure of A in B . Show that C is integrally closed in B .



14. a) For k -vector spaces V the following conditions are equivalent :

- i) Finite dimension
- ii) Finite length
- iii) a.c.c
- iv) d.c.c

Moreover, if these conditions are satisfied, length = dimension.

b) Let A be a ring in which the zero ideal is a product m_1, m_2, \dots, m_n of (not necessarily distinct) maximal ideals. Then A is Noetherian if and only if A is Artinian.

15. a) State and prove Hilbert Basis theorem.

b) If A is Noetherian, then show that $A[x_1, x_2, \dots, x_n]$ is also Noetherian.

(4×16=64)

