K22P 3319

Reg. No. : ....

Name : ....

IV Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, April 2022 (2018 Admission Onwards) MATHEMATICS MAT4C15 : Operator Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Let X be a normed space and $A \in BL(X)$. Prove that A is invertible if and only if A is bounded below and surjective.

2. Let X and Y be normed spaces and F_1 and $F_2 \in BL(X, Y)$ and $k \in K$. Show that $(F_1 + F_2)' = F_1' + F_2'$, $(kF_1)' = kF_1'$.

3. Let X and Y be Banach spaces, $F : X \to Y$ is a compact map and R(F) is closed in Y. Prove that F is of finite rank.

4. If X is an infinite dimensional normed space and $A \in CL(X)$. Prove that $0 \in \sigma_a(A)$.

5. Let H be a Hilbert space. If each (A_n) is self adjoint operator in BL(H) and $||A_n - A|| \to 0$, then prove that A is self adjoint.

6. Prove that the adjoint of Hilbert Schmidt operator on a separable Hilbert space is Hilbert Schmidt operator. **(4×4=16)**

P.T.O.

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

UNIT – I

7. a) Let X be a normed space and $A \in BL(X)$ be of finite rank. Show that $\sigma_e(A) = \sigma_a(A) = \sigma(A)$.

   b) Let X be a Banach space over K and $A \in BL(X)$. Show that $\sigma(A)$ is a compact subset of K.

8. a) Let X be a normed space and X′ is separable, prove that X is separable.

   b) Let $1 \leq p \leq \infty$ and $\frac{1}{p} + \frac{1}{q} = 1$. Show that the dual of K^n with the norm $\| \ \|_p$ is linearly isomorphic to K^n with the norm $\| \ \|_q$.

9. a) Let X be a normed space and (x_n) be a sequence in X. Then prove that (x_n) is weak convergent in X if and only if

   i) (x_n) is a bounded sequence in X and

   ii) there is some $x \in X$ such that $x'(x_n) \to x'(x)$ for every x' in some subset of X′ whose span is dense in X′.

   b) Let (x'_n) be a sequence in a normed space X. if

   i) (x'_n) is bounded and

   ii) $(x'_n(x))$ is a Cauchy sequence in K for each x in a subset of X whose span is dense in X.

   Then, prove that (x'_n) is weak* convergent in X′. Is the converse true ? Justify your answer.

UNIT – II

10. a) Let X be a reflexive normed space. Prove that every closed subspace of X is reflexive.

    b) Examine the reflexivity of $L^p([a, b])$, $1 \leq p \leq \infty$.

11. a) When a normed space X is said to be uniformly convex ?

    b) Let X be a Banach space which is uniformly convex in some equivalent norm. Then prove that X is reflexive. Is the converse true ? Justify your answer.

12. Let X be a normed space, Y be a Banach space and $F \in BL(X, Y)$, then prove that

a) $CL(X, Y)$ is a closed subspace of $BL(X, Y)$.

b) $F \in CL(X, Y)$ if and only if $F' \in CL(Y', X')$.

UNIT – III

13. Let H be a Hilbert space and $A \in BL(H)$. Then prove the following,

a) A is injective if and only if $R(A^*)$ is dense in H.

b) The closure of $R(A)$ equals $Z(A^*)^{\perp}$.

c) $R(A) = H$ if and only if A^* is bounded below.

14. Let H be a Hilbert space and $A \in BL(H)$.

a) If A is normal, x_1 and x_2 are eigenvectors of A corresponding to distinct eigenvalues, then prove that $x_1 \perp x_2$.

b) Prove that every spectral value of A is an approximate eigenvalue of A.

c) Define the numerical range of A and show that it is bounded, but not closed.

15. Let A be compact operator on non-zero Hilbert space.

a) Prove that non-zero approximate eigenvalue of A is an eigenvalue of A and the corresponding eigenspace is finite dimensional.

b) If A is self adjoint, then prove that $||A||$ or $-||A||$ is an eigenvalue of A.

(4×16=64)

K23P 0203

Reg. No. : ....

Name : ....

IV Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT4C15 : Operator Theory

Time : 3 Hours | Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Let X be a Banach space over K and let $A \in BL(X)$. Prove that $\sigma(A)$ is a compact subset of K.

2. If $x_n \xrightarrow{\omega} x$ and $y_n \xrightarrow{\omega} y$ in a normed space X and $k_n \to k$ in K, prove that $x_n + y_n \xrightarrow{\omega} x + y$ and $k_n x_n \xrightarrow{\omega} kx$.

3. Let X be a reflexive normed space. Prove that X is separable if and only if X′ is separable.

4. Let X and Y be a normed spaces and $F : X \to Y$ be linear. Prove that F is a compact map if and only if for every bounded sequence (x_n) in X, $(F(x_n))$ has a subsequence which converges in Y.

5. Let H be a Hilbert space and $A \in BL(H)$. Prove that A is normal if and only if $\| A(x) \| = \| A^*(x) \|$ for all $x \in H$.

6. Let $A \in BL(H)$. If A is compact, prove that A^* is also compact.

P.T.O.

PART – B

Answer **four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. Let X be a nonzero Banach space over C and $A \in BL(X)$. Prove that

   a) $\sigma(A)$ is non empty.

   b) $r_\sigma(A) = \inf_{n=1,2,\ldots} \|A^n\|^{\frac{1}{n}} = \lim_{n\to\infty} \|A^n\|^{\frac{1}{n}}$.

8. a) Let X be a normed space and $A \in BL(X)$ be of finite rank. Prove that $\sigma_e(A) = \sigma_a(A) = \sigma(A)$.

   b) Let X, Y and Z be normed spaces. Let $F \in BL(X, Y)$ and $G \in BL(Y, Z)$. Prove that

      i) $(GF)' = F'G'$

      ii) $\|F'\| = \|F\| = \|F''\|$ and

      iii) $F'' J_X = J_Y F$.

9. a) Let X be a normed space. If X′ is separable, prove that X is separable.

   b) Prove that $x_n \xrightarrow{\omega} x$ in l^1 if and only if $x_n \to x$ in l^1.

Unit – II

10. Let X be a normed space. Prove that X is reflexive if and only if every bounded sequence in X has a weak convergent subsequence.

11. a) Let X be a uniformly convex normed space and (x_n) be a sequence in X such that $\|x_n\| \to 1$ and $\|x_n + x_m\| \to 2$ as $m, n \to \infty$. Prove that (x_n) is a Cauchy sequence.

    b) Let X and Y be normed spaces and $F \in BL(X, Y)$. If $F \in CL(X, Y)$, prove that $F' \in CL(X, Y)$. Also show that the converse holds if Y is a Banach space.

12. Let X be a normed space and $A \in CL(X)$. Prove that dim $Z(A' - kI) = \dim Z(A - kI) < \infty$ for $0 \neq k \in K$.

Unit – III

13. a) Let H be a Hilbert space and $A \in BL(H)$. Prove that there is a unique $B \in BL(H)$ such that for all $x, y \in H$, $\langle A(x), y \rangle = \langle x, B(y) \rangle$.

    b) Let H be a Hilbert space and $A \in BL(H)$. Prove that $R(A) = H$ if and only if A^* is bounded below.

14. a) Let H be a Hilbert space and $A \in BL(H)$. Let A be self adjoint. Prove that $||A|| = \sup \{|\langle A(x), x \rangle| : x \in H, ||x|| \le 1\}$.

    b) State and prove generalized Schwarz inequality.

15. a) Let H be a Hilbert space and $A \in BL(H)$. Prove that $\sigma_e(A) \subset \sigma_a(A)$ and $\sigma(A) = \sigma_a(A) \cup \{k : \overline{k} \in \sigma_e(A^*)\}$.

    b) Let $H \neq \{0\}$ and $A \in BL(H)$ be self adjoint. Prove that

    $\{m_A, M_A\} \subset \sigma_a(A) = \sigma(A) \subset [m_A, M_A]$.

K24P 0319

Reg. No. : ....

Name : ....

IV Semester M.Sc. Degree (CBSS – Reg./Supple.-(One Time Mercy Chance)/Imp.) Examination, April 2024
(2017 Admission Onwards)
MATHEMATICS
MAT4C15 : Operator Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this Part. **Each** question carries **4** marks.

1. Let X be a normed space and $A \in BL(X)$. Show that A is invertible if and only if A is bounded below and surjective.

2. Give an example to show that not every bounded sequence in X' has a weak* convergent subsequence.

3. Let Y be a Banach space, $F_n \in CL(X, Y)$, $F \in BL(X, Y)$ and $||F_n - F|| \to 0$. Prove that $F \in CL(X, Y)$.

4. Let X be an infinite dimensional normed space and $A \in CL(X)$. Prove that $0 \in \sigma_a(A)$.

5. Let H be a Hilbert space. Consider $A, B \in BL(H)$, prove that $(A + B)^* = A^* + B^*$ and $(AB)^* = B^*A^*$.

6. Let H be a Hilbert space and $A \in BL(H)$. Prove that A is normal if and only if $||A(x)|| = ||A^*(x)||$ for all $x \in H$. **(4×4=16)**

P.T.O.

PART – B

Answer **four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. Let $X = l^p$ with the norm $\| \ \|_p$, $1 \leq p \leq \infty$. For $x = (x(1), x(2), ...) \in X$, let $C(x) = (0, x(1), x(2), ..)$. Find $\sigma(C)$, $\sigma_e(C)$ and $\sigma_a(C)$.

8. Prove that dual of l^1 is l^∞.

9. a) Let X be a Banach space, $A \in BL(X)$ and $\|A\|^p < 1$ for some positive integer p. Show that I – A is invertible and $(I - A)^{-1} = \sum_{n=0}^{\infty} A^n$.

   b) Show that $x_n \xrightarrow{w} x$ in l^1 if and only if $x_n \to x$ in l^1.

Unit – II

10. Let X be a reflexive normed space. Prove that

    a) X is Banach and it remains reflexive in any equivalent norm

    b) X′ is reflexive

    c) Every closed subspace of X is reflexive

    d) X is separable if and only if X′ is separable.

11. a) Let X be a Banach space which is uniformly convex in some equivalent norm. Prove that X is reflexive.

    b) Let X and Y be normed spaces and $F \in BL(X, Y)$. If $F \in CL(X, Y)$, show that $F' \in CL(Y', X')$.

12. Let X be a normed space and $A \in CL(X)$. Prove that every nonzero spectral value of A is an eigenvalue of A.

Unit – III

13. a) Let H be a Hilbert space and $A \in BL(H)$. Prove that $R(A) = H$ if and only if A^* is bounded below.

b) Let H be a Hilbert space and $A \in BL(H)$. Prove that A is unitary if and only if $||A(x)|| = ||x||$ for all $x \in H$ and A is surjective.

c) Give examples of positive operators A and B such that composition operators AB may not be a positive operator.

14. a) State and prove generalized Schwarz inequality.

b) Let $A \in BL(H)$. Prove that $\sigma_e(A) \subset \omega(A)$ and $\sigma(A)$ is contained in closure of $\omega(A)$.

15. a) Let $A \in BL(H)$ be normal. If x_1 and x_2 are eigenvectors of A corresponding to distinct eigenvalues, prove that $x_1 \perp x_2$.

b) Let $A \in BL(H)$ be Hilbert Schmidt operator. Prove that

i) A is compact.

ii) A^* is Hilbert Schmidt operator. **(4×16=64)**

K24P 0320

Reg. No. : ....

Name : ....

IV Semester M.Sc. Degree (C.B.S.S. – Reg./Supple. – (One Time Mercy Chance)/Imp.) Examination, April 2024
(2017 Admission Onwards)
MATHEMATICS
MAT4C16 : Differential Geometry

Time : 3 Hours Max. Marks : 80

PART – A

Answer **four** questions from this part. **Each** question carries **4** marks. **(4×4=16)**

1. Sketch the gradient vector field of the function $f(x_1, x_2) = x_1^2 + x_2^2$.
2. Sketch the graph of the function $f(x_1, x_2) = x_1^2 - x_2^2$.
3. Prove that $X \dot{+} Y = \dot{X} + \dot{Y}$.
4. Define (i) Radius of Curvature (ii) Circle of Curvature, of a plane curve C.
5. Explain why unit speed curves are parametrized by arc length.
6. Find the length of the parametrized curve $\alpha(t) = (\sin t, \cos t, \sin t, \cos t)$ in $[0, 2\pi]$.

PART – B

Answer **four** questions from this part without omitting **any** Unit. **Each** question carries **16** marks. **(4×16=64)**

Unit – I

7. a) Find the integral curve through (1, –1) of the vector field $X(x_1, x_2) = (x_1, x_2, -x_2, -\frac{1}{2} x_1)$.

   b) Show that the unit n sphere is an n surface in R^{n+1}.

   c) Sketch the typical level curves for c = –1, 0, 1 and graph of the function $f(x_1, x_2) = x_1 - x_2^2$.

P.T.O.

8. a) Prove the following : Lt $S = f^{-1}(c)$ be an n – surface in R^{n+1}, where $f : U \to R$ is such that $\nabla f(q) \neq 0$ for all $q \in S$, an let X be a smooth vector field on U whose restriction to S is a tangent vector field on S. If $\alpha : I \to U$ is any integral curve of X such that $\alpha(t_0) \in S$ for some t_0 in I, then $\alpha(t) \in S$ for all $t \in I$.

   b) Find the maximum value and minimum value of the function $g(x_1, x_2) = ax_1^2 + bx_2^2 + 2x_1x_2$, $x_1, x_2 \in R$ on the unit circle $x_1^2 + x_2^2 = 1$.

   c) Find the orientations on the cylinder $x_1^2 + x_3^2 = 1$ in R^3.

9. a) Find and sketch the gradient field of the function $f(x_1, x_2) = x_1^2 + x_2^2$.

   b) i) Verify that a surface of revolution is a 2 – surface.

   ii) Sketch the surface of revolution obtained by rotating the curve $x_2 = 2$.

   c) Show that graph of any function $f : R^n \to R$ is a level set for some function $F : R^{n+1} \to R$.

Unit – II

10. a) Prove the following : Let S be an n surface in R^3 and $\alpha : I \to S$ be a geodesic in S with $\dot{\alpha} \neq 0$. Then a vector field X tangent to S along α is parallel along α if and only if both $||X||$ and the angle between X and $\dot{\alpha}$ are constant along α.

   b) Compute the Weingarton map for the circular cylinder $x_2^2 + x_3^2 = a^2$ in R^3 $(a \neq 0)$.

11. a) Prove the following :

   i) $\nabla_\upsilon(X + Y) = \nabla_\upsilon(X) + \nabla_\upsilon(Y)$

   ii) $\nabla_\upsilon(fX) = (\nabla_\upsilon f)\, X(p) + f(p)(\nabla_\upsilon X)$

   iii) $\nabla_\upsilon(X.Y) = (\nabla_\upsilon X).Y(p) + X(p).(\nabla_\upsilon Y)$

   b) With the usual notations, prove that $L_p(\upsilon).w = L_p(w).\upsilon$, $\forall \upsilon, w \in S_p$.

   c) With the usual notations, prove that the parallel transport $P_\alpha : S_p \to S_q$ along α is a vector space isomorphism which preserves dot product.

12. a) Find the curvature of the plane curve $C = f^{-1}(0)$ oriented by the outward normal where $f(x_1, x_2) = x_2^2 - x_1$.

   b) Show that i) $D_\upsilon(fX) = (\nabla_\upsilon f)\, X(p) + f(p)D_\upsilon X$

   ii) $\nabla_\upsilon(X.Y) = (D_\upsilon X).Y(p) + X(p).(D_\upsilon Y)$

Unit – III

13. a) Prove the following : Let η be the 1 – form on $R^2 - \{0\}$ defined by $\eta = -\dfrac{x_2}{x_1^2 + x_2^2} + \dfrac{x_1}{x_1^2 + x_2^2}$. Then for $\alpha : [a, b] \to R^2 - \{0\}$ be any closed piece wise smooth parametrized curve in $R^2 - \{0\}$, $\int_\alpha \eta = 2\pi k$.

b) Find the Gaussian curvature of the surface $x_1^2 + x_2^2 - x_3 = 0$ oriented by its outward normal.

14. a) Derive the formula for Gaussian curvature of an oriented n – surface in R^{n+1}.

b) Prove the following : Let S be an n – surface in R^{n+1} and let $p \in S$. Then there exists an open set V about $p \in R^{n+1}$ and a parametrized n surface $\phi : U \to R^{n+1}$ such that ϕ is a one one map from U on to $S \cap V$.

15. a) Obtain a Torus as a parametrized surface in R^3.

b) Prove the following : Let S be an n surface in R^{n+1} and let $f : S \to R^k$. Then f is smooth if and only if $f \circ \phi : U \to R^k$ is smooth for each local parametrization $\phi : U \to S$.

c) Let V be a finite dimensional vector space with dot product and let $L : V \to V$ be a self adjoint linear transformation on V. Prove that there exist an orthonormal basis for V consisting of eigenvectors of L.

K23P 0205

Reg. No. : ....

Name : ....

IV Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT 4E01 : Commutative Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Define ideal. Give an example. Show that the set of all nilpotent elements of a commutative ring form an ideal.

2. Let A be a nonzero ring. Then show that the following are equivalent.

   i) A is a field

   ii) The only ideal in A are 0 and (1).

3. Let $A \rightarrow B$ be a ring homomorphism and let p be a prime ideal of A. Then prove that p is the contraction of a prime ideal of B if and only if $p^{ec} = p^{c}$.

4. Let q be a prime ideal in a ring A. Show that r(q) is the smallest prime ideal containing q.

5. If $x \in B$ is integral over A, show that A[x] is finitely generated A – module.

6. Let $A \subset B$ be integral domains, B integral over A. Prove that B is a field if and only if A is field.

P.T.O.

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Show that if $x \in \mathcal{R}$, the Jacbson radical if and only if 1 – xy is a unit in $A, \forall y \in A$.

   b) Show that the set $\mathcal{R}$ of all nilpotent elements in a ring A is an maximal ideal, and $A \backslash \mathcal{R}$ has no nonzero nilpotent elements.

8. a) Let $p_1, p_2, ..., p_n$ be prime ideals and let α be an ideal contained in $\bigcup_{i=1}^{n} p_i$. Then show that $\alpha \subset p_i$, for some i.

   b) Let $\alpha_1, \alpha_2, ..., \alpha_n$ be ideals and p be a prime ideal containing $\bigcap_{i=1}^{n} \alpha_i$, then prove that $p \supseteq a_i$, for some i. If $p = \bigcap \alpha_i$, then prove that $p = \alpha_i$, for some i.

9. a) If $L \supset M \supset N$ are A-modules, then show that $(L/N)/(M/N) \cong L/M$.

   b) M_1 and M_2 are sub modules of M, then show that $(M_1 + M_2)/M_1 \cong M_2/(M_1 \cap M_2)$.

Unit – II

10. a) Let M be a A – module. Then prove that the following are equivalent.

    i) M = 0

    ii) $M_p = 0$ for all prime ideals p of A

    iii) $M_m = 0$ for all maximal ideals m of A.

    b) Let q be a prime ideal in a ring A, then show that r(q) is the smallest prime ideal containing q.

11. a) State and prove first uniqueness theorem.

    b) Show that if A is absolutely flat, then every prime ideal is maximal.

12. Let S be a multiplicatively closed subset of A, and let q be a p-primary ideal. Then show that the followings

    a) If $S \cap p \neq \phi$, then $S^{-1}q = S^{-1} A$.

    b) If $S \cap p = \phi$, then $S^{-1}q$ is $S^{-1}p$– primary and its contraction in A is q.

Unit – III

13. a) Let $A \subset B$, be rings, B is integral over A, let q, q′ be prime ideals of B such that $q \subset q'$ and $q^c \subset q'^c$ say, then show that $q = q'$.

    b) Let $A \subset B$ be rings, C the integral closure of A in B. Let S be a multiplicatively closed subset of A, then prove that $S^{-1}C$ is the integral closure of $S^{-1}A$ in $S^{-1}B$.

14. a) Show that the module M is a Noetherian A – module if and only if every sub module of M is finitely generated.

    b) Show that a module M has a composition series if and only if M satisfies both chain conditions.

15. a) Show that if A is Noetherian, then the polynomial ring A [x] is also Noetherian.

    b) Show that, in a Noetherian ring A every ideal is a finite intersection of irreducible ideals.

K24P 0321

Reg. No. : ....

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IV Semester M.Sc. Degree (C.B.S.S. – Reg./Supple.-(One Time Mercy Chance)/Imp.) Examination, April 2024
(2017 Admission Onwards)
MATHEMATICS
MAT 4E01 : Commutative Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Define maximal ideal. If F is a field then show that only ideals are {0}, and F itself.
2. Let a be a nilpotent element of A. Prove that 1 + a is a unit of A. Deduce that the sum of a nilpotent element and a unit is a unit.
3. Let g : A $\to$ B be a ring homomorphism such that g(s) is a unit in B for all s $\in$ S, then show that there exists a unique ring homomorphism h : S^{-1} A $\to$ B such that g = h$\circ$f.
4. Show that if, r(a) is maximal then a is primary. In particular, the powers of a maximal ideal m are m-primary.
5. If A $\subset$ B $\subset$ C are rings and, if B is integral over A, and C is integral over B, then show that C is integral over A.
6. Show that, every finite abelian group satisfies both a.c.c. and d.c.c. **(4×4=16)**

PART – B

Answer **any four** questions from this Part without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Let A be ring and m $\neq$ (1) is and ideal of A such that every x $\in$ A – m is a unit in A. Then prove that A is local ring and m its maximal ideal.

   b) Let A be a ring and m a maximal ideal of A, such that every element 1+ x, x $\in$ m, is a unit in A, then show that A is a local ring.

P.T.O.

8. a) Show that M is finitely generated A-module if and only if M is isomorphic to a quotient of A^n, for some n > 0.

b) Let M be a finitely generated A-module and α an ideal of A contained in the Jacobson radical R of A, then show that $\alpha M = M$ implies M = 0.

9. a) Let x_i, $1 \leq i \leq n$, be elements of M whose images in M/mM form a basis of this vector space. Show that the x_i generate M.

b) Let $0 \to M_0 \to M_1 + \dots M_n \to 0$ be an exact sequence of A-modules in which all the modules M_i and the kernels of all the homomorphism belong to C. Then for any additive function λ on C show that

$$\sum_{i=0}^{n} (-1)^i \lambda(M_i) = 0 \cdot$$

Unit – II

10. a) If N and P are sub modules of an A-module M, then prove the followings.

i) $S^{-1}(N + P) = S^{-1}(N) + S^{-1}(P)$

ii) $S^{-1}(N \cap P) = S^{-1}(N) \cap S^{-1}(P)$.

b) Show that every ideal in $S^{-1}A$ is an extended ideal.

11. a) Let M be a finitely generated A-module, S a multiplicatively closed subset of A, show that $S^{-1}(\text{Ann}(M)) = \text{Ann}(S^{-1}(M))$.

b) Let q be a prime ideal in a ring A, then show that r(q) is the smallest prime ideal containing q.

12. a) State and prove first uniqueness theorem.

b) Show that, the isolated primary components (i, e., the primary components q_1 corresponding to minimal prime ideals p_1) are uniquely determined by α.

Unit – III

13. a) Show that the followings are equivalent.

i) $x \in B$ is integral over A.

ii) A[x] is a finitely generated A-module.

iii) A[x] is contained in a subring C of B such that C is a finitely generated A-module.

iv) There exists a faithful A[x]-module M which is finitely generated as an A-module.

b) Let $A \subset B$ be rings and let C be the integral closure of A in B. Show that C is integrally closed in B.

14. a) For k-vector spaces V the following conditions are equivalent :

i) Finite dimension

ii) Finite length

iii) a.c.c

iv) d.c.c

Moreover, if these conditions are satisfied, length = dimension.

b) Let A be a ring in which the zero ideal is a product $m_1, m_2, ...m_n$ of (not necessarily distinct) maximal ideals. Then A is Noetherian if and only if A is Artinian.

15. a) State and prove Hilbert Basis theorem.

b) If A is Noetherian, then show that $A[x_1, x_2, ...x_n]$ is also Noetherian.

(4×16=64)