K22P 0189

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, April 2022
(2018 Admission Onwards)
MATHEMATICS
MAT 2C 06 – Advanced Abstract Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions. **Each** question carries **4** marks.

1. Prove that $\mathbb{Z}[i]$ is an Euclidean domain.
2. Construct a field of four elements by showing $x^2 + x + 1$ is irreducible in $\mathbb{Z}_2[x]$.
3. Show that it is not always possible to construct with straight edge and compass, the side of a cube that has double the volume of original cube.
4. Show that if F is a finite field of characteristic p, then the map $\sigma_p : F \to F$ defined by $\sigma_p(a) = a^p$, for $a \in F$, is an automorphism.
5. Prove that there exists only an unique algebraic closure of a field up to isomorphism.
6. If E is a finite extension of F, Then prove that {E : F} divides [E : F]. **(4×4=16)**

PART – B

Answer **any 4** questions without omitting any Unit. **Each** question carries **16** marks.

UNIT – I

7. a) State and prove Kronecker's theorem. **8**

   b) Prove that $\mathbb{Q}(\pi) \cong \mathbb{Q}(x)$, where $\mathbb{Q}(x)$ is the field of rational numbers over $\mathbb{Q}$. **4**

   c) Prove that $\mathbb{R}[x]/\langle x^2+1 \rangle \cong \mathbb{R}(i) \cong \mathbb{C}$. **4**

8. a) Prove that if D is a UFD, then D[x] is a UFD. **8**

   b) Show that not every UFD is a PID. **3**

   c) Express $18x^2 - 12x + 48$ in $\mathbb{Q}[x]$ as a product of its content with a primitive polynomial. **5**

P.T.O.

9. a) Prove that for a Euclidean domain with Euclidean norm v, v(1) is minimal among all v (a) for non-zero $a \in D$, and also $u \in D$ is a unit if and only if, v(u) = v(1). **6**

b) Let p the an odd prime in $\mathbb{Z}$. Then prove that $p = a^2 + b^2$ for $a, b \in \mathbb{Z}$, if and only if $p \equiv 1$ (modp). **10**

UNIT – II

10. a) Prove that there exists finite of p^n elements for every prime power p^n. **8**

b) Let p be a prime and $n \in \mathbb{Z}^+$. Prove that if E and E′ are fields of order p^n, then $E \cong E'$. **8**

11. a) Find the degree and basis for $\mathbb{Q}\left(\sqrt[3]{5}, 2\right)$ and $\mathbb{Q}\left(\sqrt{2}+\sqrt{3}\right)$ over $\mathbb{Q}$. **5**

b) Prove in detail that $\mathbb{Q}\left(\sqrt{3}+\sqrt{7}\right) = \mathbb{Q}\left(\sqrt{3}, \sqrt{7}\right)$. **4**

c) Define algebraic closure of a field and prove that, a field F is algebraically closed if and only if, every non constant polynomial in F[x] factors in F[x] into linear factors. **7**

12. a) Describe the group $G\left(\mathbb{Q}\sqrt{2}, \sqrt{3} / \mathbb{Q}\right)$. **4**

b) Let F be a field and let α, β are algebraic over F. Then prove that $F(\alpha) \cong F(\beta)$ if and only if α and β are conjugates over F. **6**

c) Let $\{\sigma_i / i \in I\}$ be the collection of automorphisms of a field $\bar{F}$. Then prove that the set $E_{\{\sigma_i\}}$ of all $a \in E$ left fixed by every σ_i for $i \in I$, forms a subfield of E. **6**

UNIT – III

13. a) Prove that a finite separable extension of a field is a simple extension. **8**

b) Every finite field is perfect. **8**

14. a) Show that [E : F] = 2, then E is splitting field over F. **5**

b) Show that if $E \leq \bar{F}$, is a splitting field over F, then every irreducible polynomial in F[x] having a zero in E splits in E. **6**

c) Find the splitting field and its degree over $\mathbb{Q}$ of the polynomial $(x^2 - 2)(x^3 - 2)$ in $\mathbb{Q}$ [x]. **5**

15. a) Let K be a finite extension of degree n of a finite field F of p^r elements. Then G(K/F) is cyclic of order n and is generated by σ_{p^r}, for $\alpha \in K$, $\sigma_{p^r}(\alpha) = \alpha^{p^r}$. **8**

b) State isomorphism extension theorem. **3**

c) Let f(x) be irreducible in F[x]. Then prove that all zeros in f(x) in $\bar{F}$ has same multiplicity. **5**

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K23P 0498

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT 2C 06: Advanced Abstract Algebra

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks.

1. Find $\left[Q\left(\sqrt{2}, \sqrt{3}\right) : Q\right]$.
2. Find the primitive 5^{th} root of unity in Z_{11}.
3. Distinguish between primes and irreducibles of an integral domain.
4. Is Z[i] is an integral domain ?
5. What is the order of $G\left(Q\left(\sqrt[3]{2}\right)/Q\right)$?
6. Show that $\sqrt{1+\sqrt{5}}$ is algebraic over Q.

PART – B

Answer **4** questions without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) Prove that every PID is a UFD. **7**

   b) Prove that $Z\left[\sqrt{-5}\right]$ is an integral domain but not a UFD. **9**

8. a) State and prove Kronecker's theorem. **8**

   b) How could we construct a field of 4 elements ? **8**

P.T.O.

9. a) State and prove Gauss's Lemma. **6**

   b) An ideal <p> in a PID is maximal if and only if p is irreducible. **5**

   c) Prove that every Euclidian domain is PID. **5**

Unit – II

10. a) If α and β are constructible real numbers, then $\alpha + \beta$, $\alpha - \beta$, $\alpha\beta$ and α/β, if $\beta \neq 0$. **12**

    b) If E is a finite of characteristic P, then E contains exactly P^n elements for some positive n. **4**

11. a) Prove that trisecting an angle is impossible. **8**

    b) Prove that a finite field $GF(P^n)$ of P^n elements exists for every prime power P^n. **8**

12. a) State and prove Conjugation isomorphism theorem. **10**

    b) Define Frobenius automorphism.Also prove that $F_{\{\sigma_p\}} \cong Z_p$. **6**

Unit – III

13. a) A Field E, where $F \leq E \leq K$, is a splitting field over F if only if every automorphism of $\bar{F}$ leaving F fixed maps E onto itself and thus induces an automorphism of F leaving F fixed. **12**

    b) Let f(x) be irreducible in F[x]. Then prove that all zeros of f(x) in $\bar{F}$ have the same multiplicity. **4**

14. a) Prove that every finite field is perfect. **12**

    b) Find the splitting field of $x^3 - 2$ over Q. **4**

15. a) State the main theorem of Galois Theory. **6**

    b) State and prove Primitive Element theorem. **10**

K22P 0190

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, April 2022
(2018 Admission Onwards)
MATHEMATICS
MAT 2C 07 : Measure and Integration

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Show that if $m^*(E) = 0$, then E is measurable.
2. Show that there exists an uncountable set with measure zero.
3. Give an example of a function which is Lebesque integrable but not Riemann integrable.
4. Prove that if f and g are integrable functions, then f + g is also integrable.
5. Define $L^p(\mu)$ and prove that if f, g $\in L^p(\mu)$ and a, b are constants, then af + bg $\in L^p(\mu)$.
6. Define integral of a measurable simple function with respect to a measure μ.

(4×4=16)

PART – B

Answer **any four** questions from this Part without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) Prove that every interval is measurable.
   b) Prove that the class of all Lebesque measurable functions is a σ – algebra.
   c) Show that for any measurable function f and g.
   ess.sup. (f + g) $\leq$ ess.sup.f + ess.sup.g and give an example of strict inequality.

P.T.O.

8. a) Construct a non-measurable set.
   b) Let f be a measurable function and let f = g a.e, then prove that g is measurable.

9. a) State and prove Fatuous Lemma.
   b) Show that $\int_1^{\infty} \frac{dx}{x} = \infty$.

Unit – II

10. a) State and prove Lebesque Dominated convergence theorem.
    b) Let f be a bounded measurable function defined on the finite interval (a, b). Show that $\lim_{\beta \to \infty} \int_a^b f(x) \sin \beta x \, dx = 0$.

11. a) Let μ^* be an outer measure of H(R) and let S* denote the class of μ^* – measurable sets. Then prove that S* is a σ – ring and μ^* restricted to S* is a complete measure.
    b) Define a σ – finite measure. Show that if μ is a σ – finite measure on R, then the extension $\bar{\mu}$ of μ to S* is also σ – finite.

12. a) Show that Lebesque measure is a σ – finite measure and complete.
    b) If μ is a σ – finite measure on a ring R, then prove that it has a unique extension to the σ – ring S(R).

Unit – III

13. a) Let $[\![X, S, \mu]\!]$ be a measure space and f a non-negative measurable function. Then prove that $\varphi(E) = \int_E f \, d\mu$ is a measure on the measurable space $[\![X, S]\!]$. Also prove that if $\int f \, d\mu < \infty$, then $\forall \in > 0$, $\exists \, \delta > 0$ such that, if $A \in S$ and $\mu(A) < \delta$, then $\varphi(A) < \delta$.
    b) Define $L^{\infty}(X, \mu)$ and prove that $L^{\infty}(X, \mu)$ is a vector space over the real numbers.

14. a) State and prove Hölder's inequality.
    b) State and prove Minkowski's inequality.

15. a) If $1 \le p \le \infty$ and $\{f_n\}$ is a sequence in $L^p(\mu)$ such that $\|f_n - f_m\|_p \to 0$ as m, n $\to \infty$, then prove that there exists a function f and a subsequence $\{n_i\}$ such that $\lim f_{n_i} = f$ a.e. Also prove that $f \in L^p(\mu)$ and $\lim \|f_n - f\|_p = 0$.
    b) Prove that $L^{\infty}(\mu)$ is a complete metric space. **(4×16=64)**

K23P 0499

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT 2C 07 : Measure and Integration

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks.

1. Show that every countable set has measure zero.
2. Define measurable function. Show that every continuous functions are measurable.
3. Let f(x) is function defined on [0, 2] defined by : f(x) = 1 for x rational, if x is irrational, f(x) = –1, then find $\int_0^2 f dx$.
4. If A and B are disjoint measurable sets, then show that $\int_{A\cup B} f dx = \int_A f dx + \int_B f dx$.
5. Show that $L^{\infty}(X, \mu)$ is a vector space over the real numbers.
6. State and prove Minkowski's inequality.

PART – B

Answer **any 4** questions without omitting **any** Unit . **Each** question carries **16** marks.

Unit – I

7. a) Prove that Every interval is measurable.
   b) Define Borel sets. Show that every Borel set is measurable.
8. a) Show that collection of measurable function forms a vector space over real numbers.
   b) Show that Borel set is a proper subset of Lebesgue Measurable sets.
9. a) State and prove Fatou's Lemma.
   b) Let f and g be non-negative measurable functions. Then show that $\int f dx + \int g dx = \int (f + g) dx$.

P.T.O.

Unit – II

10. a) State and prove Lebesgue's Dominated Convergence theorem.

b) Let f be a bounded function defined on the finite interval [a, b], then prove that f is Riemann integrable over [a, b] if and only if it is continuous a.e.

11. a) Let μ^* be an outer measure on $\mathcal{H}(\mathcal{R})$ and let $\mathcal{S}^*$ denote the class of μ^* measurable sets. Then prove that $\mathcal{S}^*$ is a σ ring and μ^* restricted to $\mathcal{S}^*$ is a complete measure.

b) If μ is a σ-finite measure on a ring $\mathcal{R}$, then show that it has a unique extension to the σ-ring $\mathcal{S}(\mathcal{R})$.

12. a) Let f be bounded and measurable on a finite interval [a, b] and let $\in > 0$, then show that there exist a continuous function g such that g vanishes outside a finite interval and $\int_a^b |f - g|dx < \in$.

b) Define σ-finite and complete measure on a ring $\mathcal{R}$. Also show that Lebesgue measure m defined on $\mathcal{M}$, the class of measurable sets of $\mathbb{R}$ is σ-finite and complete.

Unit – III

13. a) Define L^p Space for $1 \leq p \leq \infty$. Also show that if $\mu(X) < \infty$ and $0 < p < q \leq \infty$ then show that $L^q(\mu) \subseteq L^p(\mu)$.

b) State and prove Holder's Inequality. When does its equality occurs ?

14. a) Let f_n be a sequence of measurable functions, $f_n : X \to [0, \infty]$, such that $f_n(x) \uparrow$ for each x and let $f = \lim f_n$ then prove that $\int f dx = \lim \int f_n d\mu$.

b) Let [[X, S, μ]] be a measure space and f a non-negative measurable function. Then prove that $\phi(E) = \int_E f d\mu$ is a measure on the measurable space [[X, S]]. Also show that if $\int f d\mu < \infty$ then $\forall \in > 0, \exists \delta > 0$ such that if $A \in S$ and $\mu(A) < \delta$, then $\phi(A) < \in$.

15. a) If $1 \leq p < \infty$ and $\{f_n\}$ is a sequence in $L^P(\mu)$ such that $||f_n - f_m||_p \to 0$ as $n, m \to \infty$ then show that there exists a function f and a sequence $\{n_i\}$ such that $\lim f_{n_i} = f$ a.e. and $f \in L^p(\mu)$.

b) Let f_n be a sequence in $L^\infty(\mu)$ such that $||f_n - f_m|| \to 0$ as $n, m \to \infty$. Then show that there exists a function f such that $\lim f_n = f$ a.e, $f \in L^\infty(\mu)$ and $\lim ||f_n - f||_\infty = 0$.

K23P 0500

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT 2C08 : Advanced Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks.

1. Let $X = \{0\} \cup \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$.

   a) Define a topology $\mathcal{T}_1$ on X such that (X, $\mathcal{T}_1$) is a compact space. Justify your answer.

   b) Define a topology $\mathcal{T}_2$ on X such that (X, $\mathcal{T}_2$) is not compact space. Justify your answer.

2. Prove or disprove : Every compact subset of a topological space is closed.

3. Prove that complete regularity is a topological property.

4. Give an example of Lindeloff space which is not compact.

5. Define Hilbert cube. Prove that a Hilbert cube is metrizable.

6. Prove that a normed space is completely regular.

P.T.O.

PART – B

Answer **any 4** questions without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Let (X, $\mathcal{T}$) be a T_1 space. Prove that X is a countably compact if and only if it has the Bollzano-Weierstrass property.

   b) Show that the condition that X is a T_1 space in part (a) is necessary. Justify your claim.

8. Prove that the product of any finite number of compact spaces is compact.

9. a) Prove or disprove : Local compactness is a topological property.

   b) Prove that every closed subspace of a locally compact Hausdorff space is locally compact.

   c) Give an example of a metric space which is locally compact but not sequentially compact.

Unit – II

10. a) Prove that every finite set in a T_1 space is closed.

    b) Prove that every second countable space is Lindeloff.

    c) Is the converse of part (b) true ? Justify your claim.

11. a) Define a completely normal topological space. Prove that a T_1 space (X, $\mathcal{T}$) is completely normal iff every subspace of X is normal.

    b) Prove that every second countable regular space is normal.

12. a) Let $\{(x_\alpha, \mathcal{T}_\alpha) : \alpha \in \Lambda\}$ be a family of topological spaces and let $X = \prod_{\alpha \in \Lambda} X_\alpha$.
    Prove that X is completely regular iff $(X_\alpha, \mathcal{T}_\alpha)$ is completely regular for each $\alpha \in \Lambda$.

    b) Let (X, $\mathcal{T}$) be a topological space with a dense subset D and a closed, relatively discrete subset C such that $P(D) \lesssim C$. Then prove that (X, $\mathcal{T}$) is not normal.

    c) Give an example of a Lindeloff space that is not separable. Justify your answer.

Unit – III

13. a) Prove that a T_1 – space $(X, \mathcal{T})$ is normal if and only if whenever A is a closed subset of X and $f : A \to [-1, 1]$ is a continuous function, then there is a continuous function $F : X \to [-1, 1]$ such that $F|_A = f$.

b) Using (a) part, state and prove Uryshon lemma.

14. State and prove Alexander sub base theorem.

15. a) State Urysohn metrization theorem. Using the Urysohn Metrization theorem prove the following :

Let (X, d) be a compact metric space, let $(Y, \mathcal{U})$ be a Hausdorff space and let $f : X \to Y$ is a continuous function that maps X onto Y. Prove that $(Y, \mathcal{U})$ is metrizable.

b) Let $(X, \mathcal{T})$ and $(Y, \mathcal{U})$ be topological spaces. Then show that homotopy $(\simeq)$ is an equivalence relation on C(X, Y), the collection of continuous functions that maps X into Y.

K23P 0501

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT2C09 : Foundations of Complex Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks.

1. Evaluate the integral $\int_{\gamma} \frac{dz}{z^2+1}$, where $\gamma(t) = 2e^{it}$, $0 \leq t \leq 2\pi$.
2. Define index of a closed rectifiable curve with respect to a point. Illustrate with example.
3. Determine the nature of the singularity of the function $f(z) = \frac{\log(z+1)}{z^2}$.
4. Find the residue of f(z) = tanz at $z = \frac{\pi}{2}$.
5. Define pole and essential singularity, giving one example of each.
6. State Arzela-Ascoli theorem in space of continuous functions.

PART – B

Answer **any 4** questions without omitting any Unit. **Each** question carries **16** marks.

Unit – I

7. a) If G is a region and $f : G \to \mathbb{C}$ is an analytic function such that there is a point a in G with $|f(a)| \geq |f(z)|$ for all z in G. Prove that f is constant.

   b) If G is simply connected and $f : G \to \mathbb{C}$ is analytic in G. Prove that f has primitive in G.

P.T.O.

8. a) Let γ be a closed rectifiable curve in $\mathbb{C}$. Prove that $n(\gamma, a)$ is constant in each component of $\mathbb{C} - \gamma$.

   b) Evaluate the integral $\int_\gamma \frac{(e^z - e^{-z})dz}{z^n}$, where n is a positive integer and $\gamma(t) = e^{it}$, $0 \le t \le 2\pi$.

9. State and prove Goursat's theorem.

Unit – II

10. a) Find the Laurent series expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ in ann(0, 1, 2).

    b) Let z = a be an isolated singularity of f and let $f(z) = \sum_{-\infty}^{\infty} a_n (z-a)^n$ be its Laurent expansion in ann(a, 0, R). Prove that z = a is a pole of order m if and only if $a_m \neq 0$ and $a_n = 0$ for $n \le -(m+1)$.

11. a) Evaluate the integral $\int_0^\pi \frac{d\theta}{a + \cos\theta}$, $a > 1$ by the method of residues.

    b) State and prove Argument principle.

12. Let $D = \{z : |z| < 1\}$ and suppose f is analytic on D with $|f(z)| \le 1$ for z in D and $f(0) = 0$. Prove that $|f'(0)| \le 1$ and $|f(z)| \le |z|$ for all z in the disk D.

Unit – III

13. a) Prove that $C(G, \Omega)$ is a complete metric space.

    b) If $\mathcal{F} \subset C(G, \Omega)$ is equicontinuous at each point of G. Prove that $\mathcal{F}$ is equicontinuous over each compact subset of G.

14. a) State and prove Hurwitz's theorem.

    b) Let $\text{Re}\, z_n > -1$, prove that the series $\sum_{n=1}^{\infty} \log(1 + z_n)$ converges absolutely if and only if the series $\sum_{n=1}^{\infty} z_n$ converges absolutely.

15. State and prove Weierstrass Factorization theorem.

K24P 0864

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (C.B.S.S. – Supple. (One Time Mercy Chance)/Imp.) Examination, April 2024
(2017 to 2022 Admissions)
MATHEMATICS
MAT 2C 09 : Foundations of Complex Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Attempt **any four** questions from this part. **Each** question carries **4** marks :

1. Given that γ and σ are closed rectifiable curves having the same initial points. Prove that $n(\gamma + \sigma, a) = n(\gamma, a) + n(\sigma, a)$ for every $a \notin \{\gamma\} \cup \{\sigma\}$.

2. Let f be analytic on B(0, 1) and suppose $|f(z)| \leq 1$ for $|z| < 1$. Show that $|f'(0)| \leq 1$.

3. Does the function $f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$ has an essential singularity at z = 0 ? Justify your answer.

4. Using residue Theorem, prove that $\int_0^\infty \frac{1}{1+x^2}\,dx = \frac{\pi}{2}$.

5. Define the set $C(G, \Omega)$ and show that it is non-empty.

6. State the Weierstrass Factorization theorem.

PART – B

Answer **any four** questions from this part without omitting any Unit. **Each** question carries **16** marks :

Unit – I

7. a) Prove the following : If G is simply connected and $f : G \to C$ is analytic in G then f has a primitive in G.

   b) State and prove The Open Mapping Theorem.

P.T.O.

8. State and prove the Third Version of Cauchy's Theorem.

9. Prove the following : let G be a connected open set and let $f : G \to C$ be an analytic function. Then the following conditions are equivalent.

   a) $f \equiv 0$;

   b) there is a point a in G such that $f^n(a) = 0$ for each $n \geq 0$;

   c) $\{z \in G : f(z) = 0\}$ has a limit point in G.

Unit – II

10. a) Show that for a > 1, Show that $\int_0^{\pi} \frac{d\theta}{a + \cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}}$.

    b) State and prove the Residue theorem.

11. State and prove the Laurent Series Development.

12. Prove the following :

    a) If $|a| < 1$ then $\varphi_a(z) = \frac{z-a}{1-\bar{a}z}$ is a one-one map of $D = \{z : |z| < 1\}$ on to itself ; the inverse of φ_a is φ_{-a}. Furthermore, φ_a maps ∂D on to ∂D, $\varphi'_a(0) = 1 - |a|^2$ and $\varphi'_a(a) = (1 - |a|^2)^{-1}$.

    b) Let $f(z) = \frac{1}{z(z-1)(z-2)}$; give the Laurent series of f(z) in each of the following annuli :

       i) ann(0 ; 0, 1),

       ii) ann (0 ; 1, 2),

       iii) ann (0 ; 2, ∞).

Unit – III

13. a) Prove the following : If G is open in C then there is a sequence $\{K_n\}$ of compact subsets of G such that $G = \cup_{n=1}^{\infty} K_n$. Moreover the sets K_n can be chosen to satisfy the following conditions :

       i) $k_n \subset \text{int } K_{n+1}$.

       ii) $K \subset G$ and K is compact implies $K \subset K_n$ for some n.

       iii) Every component of $C_\infty - K_n$ contains a component of $C_\infty - G$.

    b) State and prove Hurwitz's theorem.

14. a) With the usual notations, prove that $|1 - E_p(z)| \le |z|^{p+1}$ for $|z| \le 1$ and $p \ge 0$.

b) Discuss the convergence of the infinite product $\prod_{n=1}^{\infty} \frac{1}{n^p}$ for $p > 0$.

15. a) Show that $\prod(1+z_n)$ converges absolutely iff $\prod(1+|z_n|)$ converges.

b) Prove the following : If $Re z_n > 0$ then the product $\prod z_n$ converges absolutely iff the series $\Sigma(z_n - 1)$ converges absolutely.

c) Prove the following : Let $Re z_n > 0$ for all $n \ge 1$. Then $\prod_{n=1}^{\infty} z_n$ converges to a non zero number iff the series $\sum_{n=1}^{\infty} \log z_n$ converges.

K22P 0193

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, April 2022
(2018 Admission Onwards)
MATHEMATICS
MAT 2C10 : Partial Differential Equations and Integral Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Eliminate the arbitrary function F from $z = F\left(\frac{xy}{z}\right)$ and find the corresponding partial differential equation.

2. Find the general solution of yzp + xzq = xy.

3. Show that the solution of the Dirichlet problem if it exists is unique.

4. Find the Riemann function of the equation $Lu = u_{xy} + \frac{1}{4}u = 0$.

5. Transform the problem $y'' + xy = 1$, $y(0) = 0$, $y(l) = 1$ into an integral equation.

6. Prove that the characteristic numbers of a Fredholm equation with a real symmetric kernel are all real. **(4×4=16)**

PART – B

Answer **four** questions from this Part, without omitting **any** Unit. **Each** question carries **16** marks.

Unit – 1

7. a) Show that the Pfaffian differential equation $\vec{X}.\vec{dr} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ is integrable if and only if $\vec{X}.\text{curl}\,\vec{X} = 0$.

   b) Show that ydx + xdy + 2zdz = 0 is integrable and find its integral.

P.T.O.

8. a) Find a complete integral of $z^2 - pqxy = 0$ by Charpits method.

b) Solve $u_x^2 + u_y^2 + u_z = 1$ by Jacobi's method.

9. a) Find a complete integral of the equation $(p^2 + q^2)\, x = pz$ and the integral surface containing the curve $C : x_0 = 0,\ y_0 = s^2,\ z_0 = 2s$.

b) Solve $xz_y - yz_x = z$ with the initial condition $z(x, 0) = f(x),\ x \geq 0$.

Unit – 2

10. a) Reduce the equation $u_{xx} + 2u_{xy} + 17u_{yy} = 0$ into canonical form.

b) Derive d'Alembert's solution of one dimensional wave equation.

11. a) Solve $y_{tt} - C^2 y_{xx} = 0,\ 0 < x < 1,\ t > 0$.

$y(0, t) = y(1, t) = 0$

$y(x, 0) = x(1 - x),\ 0 \leq x \leq 1$

$y_t(x, 0) = 0,\ 0 \leq x \leq 1$

b) State and prove Harnack's theorem.

12. a) Solve the differential equation corresponding to heat conduction in a finite rod.

b) Prove that the solution $u(x, t)$ of the differential equation

$u_t - ku_{xx} = F(x, t),\ 0 < x < l,\ t > 0$ satisfying the initial condition

$u(x, 0) = f(x),\ 0 \leq x \leq l$ and the boundary conditions

$u(0, t) = u(l, t) = 0,\ t \geq 0$ is unique.

Unit – 3

13. a) Solve $y'' = f(x),\ y(0) = y(l) = 0$.

b) Solve $x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} + (\lambda x^2 - 1)y = 0$ with $y(0) = 0,\ y(1) = 0$.

14. a) If y_m, y_n are characteristic functions corresponding to different characteristic numbers λ_m, λ_n of $y(x) = \lambda\int_0^1 K(x,\xi)y(\xi)\,d\xi$, then if K (x, ξ) is symmetric. Prove that y_m and y_n are orthogonal over (a, b).

b) Solve the integral equation $y(x) = f(x) + \lambda\int_0^1 (1-3x\xi)y(\xi)\,d\xi$ and discuss all its possible cases.

15. a) Describe the iterative method for solving Fredholm equation of second kind.

b) Find the iterated Kernels $K_2(x, \xi)$ and $K_3(x, \xi)$ associated with $K(x, \xi) = |x - \xi|$ in the interval [0, 1]. **(4×16=64)**

K23P 0502

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.)
Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT 2C 10 : Partial Differential Equations and Integral Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks.

1. Eliminate the arbitrary function F from the equation $F(z - xy, x^2 + y^2)$ and find the corresponding Partial differential equation.
2. Show that $z = ax + \frac{y}{a} + b$ is complete integral of $pq = 1$.
3. State and prove maximum principle for harmonic function.
4. Prove that the solution of Neumann problem is unique up to the addition of a constant.
5. Define Fredholm integral equation of second kind and give an example.
6. Solve the integral equation $y(x) = 1 + \lambda\int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi$ by iterative method.

PART – B

Answer **any 4** questions without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Find the general integral of the equation
$y^2p - xyq = x(z - 2y)$.
b) Prove that the equations $f = xp - yq - x = 0$, $g = x^2p + q - xz = 0$ are compatible and find a one parameter family of common solutions.

8. a) Find the complete integral of $(p^2 + q^2)y - qz = 0$ by Charpit's method.
b) Solve the PDE by Jacobi's method
$z^2 + zu_x - u_x^2 - u_y^2 = 0$.

P.T.O.

9. a) Find the integral surface of the equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ which passes through the line $x_0(s) = 1$, $y_0(s) = 0$ and $z_0(s) = s$.

b) Find the characteristic strips of the equation $xp + yq = pq$ where the initial curve is $c : z = \frac{x}{2}, y = 0$.

Unit – II

10. a) Reduce the equation $u_{xx} - 4x^2 u_{yy} = \frac{1}{x} u_x$ into a canonical form.

b) Derive d′ Alemberts solution of wave equation.

11. a) Solve $y_{tt} - c^2 y_{xx} = 0$, $0 < x < 1$, $t > 0$

$y(0, t) = y(1, t) = 0$

$y(x, 0) = x(1 - x)$, $0 \le x \le 1$

$y_t(0, t) = 0$, $0 \le x \le 1$

b) Prove that solution of Dirichlet problem is stable.

12. a) Solve the Dirichlet problem for a circle.

b) Solve the heat conduction problem in a finite rod.

Unit – III

13. a) Transform the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0$, $y(0) = 0$, $y(l) = 0$ to an integral equation.

b) Show that the characteristic function of the symmetric Kernel corresponding to distinct characteristic numbers are orthogonal.

14. a) Using Green's function, solve the boundary value problem
$y'' + xy = 1$, $y(0) = 0$, $y(l) = 1$.

b) Show that any solution of the integral equation $y(x) = \lambda \int_0^1 (1 - 3xy) y(\xi) d\xi + F(x)$ can be expressed as the sum of F(x) and some linear combination of the characteristic functions.

15. a) Show that the integral equation $y(x) = 1 + \frac{1}{\pi} \int_0^{2\pi} \sin(x + \xi) y(\xi) d\xi$ possess infinitely many solution.

b) Find the Resolvent Kernel for the Kernel $k(x, \xi) = xe^{\xi}$ in the interval [– 1, 1].

K22P 3322

Reg. No. : ....

Name : ....

IV Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, April 2022
(2018 Admission Onwards)
MATHEMATICS
MAT 4E02 : Fourier and Wavelet Analysis

Time : 3 Hours   Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** carries **4** Marks.

1. Define the conjugate reflection of $\omega \in l^2(Z_N)$. For any z, $\omega \in l^2(Z_N)$ and $k \in Z$, prove that $z * \tilde{\omega}(k) = \langle z, R_k\omega \rangle$.

2. Explain downsampling operator and upsampling operator.

3. If N = 2M for some natural number M, $z \in l^2(Z_N)$ and $\omega \in l^2(Z_{N/2})$, then prove that $D(z)*\omega = D(z*U(\omega))$.

4. If $z \in l^2(Z)$ and $\omega \in l^1(Z)$, show that $z* \omega \in l^2(Z)$ and $||z*\omega|| \le ||\omega||_1||z||$.

5. Define translation-invariant linear transformation on $l^2(Z)$. If $T : l^2(Z) \to l^2(Z)$ is bounded and translation-invariant, then show that $T(z) = b*z$ for all $z \in l^2(Z)$, where $b = T(\delta)$.

6. If $z \in l^2(Z)$, show that $(z^*)\text{^}(\theta) = z\text{^}(\theta + \pi)$.

7. If $f,g \in L^1(R)$, show that $f*g \in L^1(R)$ and $||f*g||_1 \le ||f||_1||g||_1$.

8. If $f,g \in L^1(R)$, and if $\hat{f}, \hat{g} \in L^1(R)$, prove that $\langle \hat{f}, \hat{g} \rangle = 2\pi \langle f, g \rangle$. **(4×4=16)**

P.T.O.

PART – B

Answer **any four** questions from this part, without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

9. a) Let $w \in l^2(Z_N)$. Then show that $\{R_k w\}_{k=0}^{N-1}$ is an orthonormal basis for $l^2(Z_N)$ if and only if $|\hat{w}(n)| = 1$ for all $n \in Z_N$.

   b) Suppose M is a natural number, N = 2M and $z \in l^2(Z_N)$. Define $z* \in l^2(Z_N)$ by $z*(n) = (-1)^n z(n)$ for all n. Then show that $(z*)\hat{}(n) = \hat{z}(n + M)$ for all n.

10. Suppose M is a natural number and N = 2M. If $u, v \in l^2(Z_N)$, show that $\{R_{2k}v\}_{k=0}^{M-1} \cup \{R_{2k}u\}_{k=0}^{M-1}$ is an orthonormal basis for $l^2(Z_N)$ if and only if the system matrix A(n) of u, v is unitary for each n = 0, 1, ... , M – 1.

11. Suppose M is a natural number, N = 2M and $u,v,s,t \in l^2(Z_N)$. Show that $\tilde{t} * U(D(z * \tilde{v})) + \tilde{s} * U(D(z * \tilde{u})) = z$ for all $z \in l^2(Z_N)$ if and only if $A(n)\begin{bmatrix}\hat{s}(n)\\ \hat{t}(n)\end{bmatrix} = \begin{bmatrix}\sqrt{2}\\ 0\end{bmatrix}$ for each n = 0, 1, ... ,N –1, where A(n) is the system matrix of u, v.

12. If $2^p|N$, explain the construction of a p^{th} stage wavelet basis for $l^2(Z_N)$ from a given p^{th} stage wavelet filter sequence.

Unit – II

13. a) If $\{a_j\}_{j\in Z}$ is an orthonormal set in a Hilbert space H, and if $f \in H$, show that the sequence $\{\langle f, a_j\rangle\}_{j\in Z} \in l^2(Z)$.

    b) Show that an orthonormal set $\{a_j\}_{j\in Z}$ in a Hilbert space H is a complete orthonormal set if and only if $f = \sum_{j\in Z} \langle f, a_j \rangle a_j$ for all f in H.

14. a) Suppose $f \in L^1([-\pi, \pi))$ and $\langle f, e^{in\theta}\rangle = 0$ for all $n \in Z$, show that $f(\theta) = 0$ a.e.

    b) If $z \in l^2(Z)$ and $\omega \in l^1(Z)$, prove that $(z*\omega)\hat{}(\theta) = \hat{z}(\theta)\,\hat{w}(\theta)$ a.e.

15. If $T : L^2([-\pi, \pi)) \to L^2([-\pi, \pi))$ is a bounded, translation-invarient linear transformation, then show that there exists $\lambda_m \in C$ such that $T(e^{im\theta}) = \lambda_m e^{im\theta}$ for each $m \in Z$.

16. Suppose that $u,v \in l^1(Z)$. Show that $B = \{R_{2k}v\}_{k\in Z} \cup \{R_{2k}u\}_{k\in Z}$ is a complete orthonormal set in $l^2(Z)$ if and only if the system matrix $A(\theta)$ is unitary for all $\theta \in [0, \pi)$.

Unit – III

17. Define approximate identity. Suppose $f \in L^1(R)$ and $\{g_t\}_{t>0}$ is an approximate identity. Then show that for every Lebesgue point x of f, $\lim_{t \to 0^+} g_t * f(x) = f(x)$.

18. Define Fourier transform and inverse Fourier transform on R. Suppose $f \in L^1(R)$ and $\hat{f} \in L^1(R)$, then show that $\frac{1}{2\pi}\int_R \hat{f}(\xi)e^{ix\xi}\, d\xi = f(x)$ a.e. on R. Use this to establish the uniqueness of Fourier transform.

19. Suppose $f \in L^2(R)$, $\{f_n\}_{n=1}^{\infty}$ is a sequence of functions such that $f_n, \hat{f}_n \in L^1(R)$ for each n, and $f_n \to f$ in $L^2(R)$ as $n \to \infty$. Show that $\{\hat{f}_n\}_{n=1}^{\infty}$ converges to a unique $F \in L^2(R)$. Also, show that if $f \in L^1(R) \cap L^2(R)$, then $F = \hat{f}$.

20. If f, $g \in L^2(R)$, prove that $\langle \hat{f}, \hat{g} \rangle = 2\pi \langle f, g \rangle$ and $\|\hat{f}\| = \sqrt{2\pi}\|f\|$. If $f \in L^2(R)$ and if $\{f_n\}_{n=1}^{\infty}$ is a sequence of L^2 – functions such that $f_n \to f$ in $L^2(R)$ as $n \to \infty$, then prove that $\hat{f}_n \to \hat{f}$ in $L^2(R)$ as $n \to \infty$. **(4×16=64)**

K24P 1104

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (C.B.C.S.S. – OBE – Regular)
Examination, April 2024
(2023 Admission)
MATHEMATICS
MSMAT02C07/MSMAF02C07 : Measure Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any five** questions from this Part. **Each** question carries **4** marks. **(5×4=20)**

1. Define Leabesgue outer measure. Show that $m^*(A) \leq m^*(B)$ if $A \subseteq B$.

2. Show that every countable set has measure zero.

3. Show that if f is a non-negative measurable function, then f = 0 a.e. if and only if $\int f dx = 0$.

4. Prove that if f and g are measurable. $|f| \leq |g|$ a.e., and g is integrable, then f is integrable.

5. Prove that every algebra is a ring, but that the converse is not true.

6. Define $L^p(\mu)$. Let $f, g \in L^p(\mu)$ and let a, b be constants. Then show that $af + bg \in L^p(\mu)$.

PART – B

Answer **any three** questions from this Part. **Each** question carries **7** marks. **(3×7=21)**

7. Show that there exists uncountable sets of zero measure.

8. Let E be a measurable set. Then prove that for every y the set $E + y = [x + y : x \in E]$ is measurable and the measures are the same.

P.T.O.

9. Show that $\int_1^{\infty} \frac{dx}{x} = \infty$.

10. If μ is a measure on a ring R and if the function μ^* is defined on H(R) by $\mu^*(E) = \inf\left[\sum_{n=1}^{\infty} \mu(E_n) : E_n \in R, n = 1, 2, ..., E \subseteq \bigcup_{n=1}^{\infty} E_n\right]$, then prove that
    i) for $E \in R$, $\mu^*(E) = \mu(E)$
    ii) μ^* is an outer measure on H(R).

11. Let [[X, S, μ]] be a measure space and f a non negative measurable function. Then prove that $\phi(E) = \int_E f d\mu$ is a measure on the measurable space [[X, S]]. Also prove that, if $\int f d\mu < \infty$ then $\forall \epsilon > 0, \exists \delta > 0$ such that, if $A \in S$ and $\mu(A) < \delta$, then $\phi(A) < \epsilon$.

PART – C

Answer **any three** questions from this Part. **Each** question carries **13** marks. **(3×13=39)**

12. State and prove Fatou's Lemma.

13. Show that the class M of Lebesgue measurable sets is a σ-algebra.

14. Let $\{E_i\}$ be a sequence of measurable sets. Show that
    a) if $E_1 \subseteq E_2 \subseteq ...$, then $m(\lim E_i) = \lim m(E_i)$
    b) if $E_1 \supseteq E_2 \supseteq ...$ and $m(E_i) < \infty$ for each i, then $m(\lim E_i) = \lim m(E_i)$.

15. If μ is a σ-finite measure on a ring R, then prove that it has a unique extension to the σ-ring S(R).

16. State and prove Holder's inequality. When does the equality occur ? Justify.

K24P 1106

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (CBCSS – OBE – Regular) Examination, April 2024
(2023 Admission)
MATHEMATICS
MSMAT02C08 : Advanced Real Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **five** questions from this Part. **Each** question carries **4** marks. **(5×4=20)**

1. Define pointwise convergence of a sequence of functions. For

   m = 1, 2, ... n = 1, 2, 3, ... let $s_{m,n} = \frac{m}{m+n}$. Find $\lim\limits_{n\to\infty}\lim\limits_{m\to\infty} s_{m,n}$.

2. Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

3. Let $E(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!}$. Prove that $E(z + w) = E(z) + E(w)$, $z, w \in \mathbb{C}$.

4. If f is continuous with period 2π and if $\in > 0$, prove that there is a trigonometric polynomial p such that $|p(x) - f(x)| < \in$ for all real x.

5. Define norm of a linear transformation. If $A, B \in L(\mathbb{R}^n, \mathbb{R}^m)$, then prove that $||A + B|| \le ||A|| + ||B||$.

6. If $f(0, 0) = 0$ and $f(x, y) = \frac{xy}{x^2 + y^2}$ if $(x, y) \ne (0, 0)$, find $D_1f(0, 0)$ and $D_2f(0, 0)$.

PART – B

Answer **three** questions from this Part. **Each** question carries **7** marks. **(3×7=21)**

7. State and prove Cauchy criterion for uniform convergence of a sequence of functions.

8. Prove that there exists a real valued function on the real line which is nowhere differentiable.

9. If K is compact, $f_n \in \mathscr{C}(K)$ for n = 1, 2, 3, ... and if $\{f_n\}$ is pointwise bounded and equicontinuous on K, then prove that $\{f_n\}$ contains a uniformly convergent subsequence.

10. Suppose $a_0, a_1, ..., a_n$, are complex numbers, $n \geq 1$, $a_n \neq 0$, $p(z) = \sum_0^n a_k z^k$. Then prove that p(z) = 0 for complex number z.

11. Let X be a complex metric space and if ϕ is a contraction of X into X. Prove that there exists one and only one $x \in X$ such that $\phi(x) = x$.

PART – C

Answer **three** questions from this Part. **Each** question carries **13** marks. **(3×13=39)**

12. a) Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t) = A_n$, n = 1, 2, 3,... . Then prove that $\{A_n\}$ converges and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$.

    b) Give an example of a series of continuous functions with a discontinuous sum.

13. Suppose $\{f_n\}$ is a sequence of functions differentiable on [a, b] and that $\{f_n(x_0)\}$ converges for some point x_0 on [a, b]. If $\{f_n'\}$ converges uniformly on [a, b] then prove that $\{f_n\}$ converges uniformly on [a, b] to a function f and $f'(x) = \lim_{n \to \infty} f_n'(x)$, $a \leq x \leq b$.

14. a) Define the following terms :

    i) algebra.

    ii) uniformly closed algebra.

    iii) uniform closure of an algebra.

    b) Let $\mathscr{B}$ be the uniform closure of an algebra $\mathscr{A}$ of bounded functions, then prove that $\mathscr{B}$ is a uniformly closed algebra.

15. State and prove Parseval's theorem.

16. State and prove inverse function theorem.

K24P 1107

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree (C.B.C.S.S. – OBE – Regular)
Examination, April 2024
(2023 Admission)
MATHEMATICS
MSMAT02C09/MSMAF02C09 : Advanced Topology

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 5** questions from the following 6 questions. **Each** question carries **4** marks.

1. Show that the set of integers is not well-ordered in the usual order.
2. Does the set of rationals Q is compact ? Justify your answer.
3. Show that the real line R has a countable basis.
4. Let f, g : X $\rightarrow$ Y be continuous; assume that Y is Hausdorff. Show that {x, f(x) = g(x)} is closed in X.
5. Give an example showing that a Hausdorff space with a countable basis need not be metrizable.
6. Show that the unit circle S^1 is a one-point compactification of the unit interval (0, 1). **(5×4=20)**

PART – B

Answer **any 3** questions from the following 5 questions. **Each** question carries **7** marks.

7. Prove the following : Every nonempty finite ordered set has the order type of a section {1, 2, ..., n} of Z_+, so it is well-ordered.
8. Prove that every metrizable space is normal.

P.T.O.

9. a) Prove that the product of two Lindelof space need not be Lindelof.

   b) Prove that a subspace of a Lindelof space need not be Lindelof.

10. Show that every locally compact Hausdorff space is regular.

11. Prove the following : Let $A \subset X$; let $f : A \to Z$ be a continuous map of A in to the Hausdorff space Z. There is at most one extension of f to a continuous function $g : \bar{A} \to Z$. **(3×7=21)**

PART – C

Answer **any 3** questions from the following 5 questions. **Each** question carries **13** marks.

12. Prove the following :

    a) Every closed subspace of a compact space is compact.

    b) Every compact subspace of a Hausdorff space is closed.

    c) The image of a compact space under a continuous map is compact.

13. Prove the following :

    a) A subspace of a Hausdorff space is Hausdorff.

    b) A product of Hausdorff space is Hausdorff.

    c) A product of regular space is regular.

14. State and prove The Urysohn lemma.

15. State and prove Tietze Extension Theorem.

16. State and prove Tychonoff Theorem. **(3×13=39)**