1. The number of an integer $n \times n$ matrix with determinant 1 is $c_n = \frac{1}{n+1}\binom{2n}{n}$, $n \geq 1$.

2. If sum of the elements in each column (row) of a square matrix of order n is k, then the sum of the elements in each column (row) of A^m will be k^m.

3. Let A be an $m \times n$ matrix with all entries are distinct. Then the number of sub matrix of order $i \times j$ is $\binom{m}{i} \times \binom{n}{j}$.

4. The number of sub matrix of all order of a $m \times n$ matrix A with all entries are distinct is $\sum_{i=1}^{m}\sum_{j=1}^{n}\binom{m}{i} \times \binom{n}{j} = (2^m - 1)(2^n - 1)$.

5. Let A be an $m \times n$ matrix with all entries are equal. Then the number of sub matrix of order $i \times j$ is 1.

6. The number of sub matrix of all order of a $m \times n$ matrix A with all entries are equal is nm.

7. Let V be a finite-dimensional vector space, and let $T : V \to V$ be linear.

   (i) If $rank(T) = rank(T^2)$, then $R(T) \cap N(T) = \{0\}$ and $V = R(T) \oplus N(T)$.

   (ii) $V = R(T^k) \oplus N(T^k)$ for some $k > 0$.

8. Let $\Delta_{2n} = \begin{pmatrix} a & 0 & \cdots & 0 & b \\ 0 & a & \cdots & b & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ b & 0 & \cdots & 0 & a \end{pmatrix}_{2n \times 2n}$. Then $det(\Delta_{2n}) = (a^2 - b^2)^n$.

9. Let $A \in M_{m \times n}(F)$. Then $rank(A^*A) = rank(A)$.

10. If A is an $m \times n$ matrix such that $rank(A) = n$, then A^*A is invertible.

11. For any square matrix A, $\| A \|$ is finite and, in fact, equals $\sqrt{\lambda}$, where λ is the largest eigenvalue of A^*A.

12. Let A be an invertible matrix. Then $\| A^{-1} \| = \frac{1}{\sqrt{\lambda}}$, where λ is the smallest eigenvalue of A^*A.

13. Let T be a linear operator on V whose characteristic polynomial splits, and let $\lambda_1, \lambda_2, \cdots, \lambda_k$ be the distinct eigenvalues of T. Then T is diagonalisable **iff** $rank(T - \lambda_i I) = rank(T - \lambda_i I)^2$ for $1 \leq i \leq k$.

14. A linear operator T on a finite-dimensional vector space V is diagonalisable **iff** its Jordan canonical form is a diagonal matrix.

15. The system of linear equations $Ax = b$ has a solution **iff** $b \in R(A)$ (Row space of A).

16. An orthonormal set in an inner product space is linearly independent.

17. Let W_1 and W_2 be subspaces of a finite dimensional inner product space V. Then
(i) $(W_1 + W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$
(ii)$(W_1 \cap W_2)^{\perp} = W_1^{\perp} + W_2^{\perp}$

18. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then A is diagonalisable **iff**
$(a + d)^2 > 4(ad - bc)$.

19. For any square matrix $A = [a_{ij}]$ of order n,
$R(A) = \max\{R_i(A) = \sum_{j=1}^{n} | a_{ij} | : 1 \leq i \leq n\}$
$C(A) = \max\{C_j(A) = \sum_{i=1}^{n} | a_{ij} | : 1 \leq j \leq n\}$
$s_i = R_i(A) - | a_{ii} |$

20. For any square matrix A of order n, every eigenvalue λ of A satisfies $| \lambda - a_{ll} | \leq s_l$ for some $1 \leq l \leq n$.

21. For any square matrix A of order n, every eigenvalue of A satisfies $| \lambda | \leq \min\{R(A), C(A)\}$.

22. The eigenvalue of the tridiagonal matrix $A = \begin{bmatrix} a & b_1 & 0 \cdots & \cdots & 0 \\ c_1 & a & b_2 & \cdots & \\ & c_2 & a & b_3 & \\ \vdots & \cdots & \cdots & a & b_{n-1} \\ & \cdots & & c_{n-1} & a \end{bmatrix}_{n \times n}$
$(n \geq 3)$ satisfies the inequality $| \lambda - a | < 2\sqrt{\max_i | b_i | \max_i | c_i |}$.

23. If A is idempotent. then $Rank(A) = Trac(A)$.

24. If A is symmetric matrix, then $Rank(A)$ is equal to the number of non-zero eigenvalues of A.

25. Test for diagonalization

    Let T be a linear operator on an n-dimensional vector space V. Then T is diagonalizable **iff**

    (i) The characteristic polynomial of T splits.

    (ii) For each eigenvalue λ of T, the multiplicity of λ equals

    $n - rank(T - \lambda I)$.