ALGEBRA NET MATERIAL

1. Let $GL(n,q)$ be the group of all $n\times n$ invertible matrices over the finite field $\mathbb{F}_q$, where $q=p^m$, p is a prime and some positive integer m. Then
   $(i) O(GL(n,q)) = q^{\frac{n(n-1)}{2}}(q^n-1)(q^{n-1}-1)\cdots(q-1)$.
   (ii) Order of Sylow p-subgroup of $GL(n,q)$ is $q^{\frac{n(n-1)}{2}}$.
   (iii) $O(SL(n,q)) = \dfrac{q^{\frac{n(n-1)}{2}}(q^n-1)(q^{n-1}-1)\cdots(q-1)}{(q-1)}$.
2. The center $Z(GL(n,q)) = \{kI_n \mid k\in\mathbb{F}_q^*\}$, where I_n is the identity matrix.
3. The centralizer (normalizer)
   $$N(SL(n,q)) = \{kI_n \mid k\in\mathbb{F}_q^* \text{ and } k^n=1\}.$$
4. $Z(SL(n,q)) = \{kI_n \mid k\in\mathbb{F}_q^* \text{ and } k^n=1\}$.
5. $\mid Z(SL(n,q)) \mid = gcd(n,q-1)$.
6. Let G be a finite group and $a\in G$. Then $O(cl(a)) = \dfrac{O(G)}{O(N(a))}$.
7. $\dfrac{G}{Z(G)} \cong I(G)$.
8. In S_n, the number of distinct cycles of length r is $\dfrac{n!}{r(n-r)!}$ $(r\leqslant n)$.
9. Converse of Lagrange's theorem holds in finite cyclic groups and prime power order.
10. $Z(G) = \bigcap_{a\in G} N(a)$.
11. The number of group homomorphism from $\mathbb{Z}_m$ to $\mathbb{Z}_n$ is $gcd(m,n)$.
12. Let G be an infinite cyclic group. Then $\mid Aut(G) \mid = 2$.
13. Let G be a finite cyclic group of order n. Then $\mid Aut(G) \mid = \varphi(n)$.
14. $Aut(S_n) \cong S_n$, $Z(S_n) = \{I\}$ for $n\geq 3$ and $n\neq 6$.
15. $Aut(\underbrace{\mathbb{Z}\oplus\mathbb{Z}\oplus\cdots\oplus\mathbb{Z}}_{n \text{ times}}) \cong GL(n,\mathbb{Z})$.

16. $Aut(\underbrace{\mathbb{Z}_{p^m} \oplus \mathbb{Z}_{p^m} \oplus \cdots \oplus \mathbb{Z}_{p^m}}_{n \text{ times}}) \cong GL(n, \mathbb{Z}_{p^m})$.

17. $Aut(\mathbb{Z}_2 \oplus \mathbb{Z}_4) \cong D_8$.

18. If G has order $n > 1$, then $|Aut(G)| \leq \prod_{i=0}^{k} (n - 2^i)$ where $k = [\log_2(n - 1)]$.

19. Let p be a prime number, and let G be a finite abelian group in which every element has order p. Then $Aut(G) \cong GL(n, \mathbb{Z}_p)$, where n is the dimension of G over $\mathbb{Z}_p$.

20. If G is a group of order n and F is any field, then $GL(n, F)$ contains a subgroup isomorphic to G.

21. Let $G = G_1 \times G_2 \times \cdots \times G_n$, where the G_i are abelian groups. Then $Aut(G)$ is isomorphic with the group of all invertible $n \times n$ matrices whose (i, j) entries belong to $Hom(G_i, G_j)$, the usual matrix product begin the group operation.

22. If $O(G) = p^2q^2$ and $q \nmid p^2 - 1$, $p \nmid q^2 - 1$, then G is abelian.

23. G is a finite group of order p^2q where p and q are distinct primes such that $p^2 \not\equiv 1(modq)$ and $q \not\equiv 1(modp)$. Then G is an abelian group. If p divides $q - 1$, then any group of order p^2q is abelian.

24. If p does not divide $\mid Aut(G) \mid$, then any group of order pq^2 is abelian.

25. If G is a non-abelian finite group, then $|Z(G)| \leq \frac{1}{4}|G|$.

26. If H and K are subgroups of a finite group G satisfying $(|G : H|, |G : K|) = 1$, then $G = HK$.

27. If G is a simple group of order 60, then G is isomorphic to A_5.

28. Let G be a group of order pqr, where $p > q > r$ are primes. If $p - 1$ is not divisible by q or r and $q - 1$ is not divisible by r, then G must be abelian (hence cyclic).

29. Abelian groups have exactly one Sylow p-subgroup for each p.

30. The class equation of G is
$$O(G) = O(Z(G)) + \sum_{a \notin Z(G)} \frac{O(G)}{O(N(a))}.$$

31. Let G be a non-abelian group of order p^3. The number of conjugate classes of G is $p^2 + p - 1$.

32. Let G be a finite group of order n and p be a prime number such that $p > \frac{n}{p}$. Then any subgroup of order p in G is normal in G.

33. Let G be a finite group of order n and p be a prime number such that p^2 does not divide n. Then any subgroup of order p in G is normal in G.

34. The number of non-isomorphic abelian groups of order p^n, (p a prime) is $p(n)$ (partition of n).

35. The number of groups of order n is at most $n^{n log_2 n}$.