K23U 0513

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS-OBE-Regular/Supplementary/ Improvement) Examination, April 2023
(2019 and 2020 Admissions)
CORE COURSE IN MATHEMATICS
6B10 MAT : Real Analysis – II

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. State second form of the fundamental theorem of integral calculus.
2. State Lebesgue's integrability criterion.
3. Evaluate $\int_0^\infty \frac{dx}{1+x^2}$.
4. Evaluate $\int_0^\infty x^4 e^{-x}\, dx$.
5. Find the limit of the sequence of function $f_n(x) = x^n$ on [0, 1].

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Prove that $f(x) = \sqrt{x}$ is uniformly continuous on $[1, \infty)$.
7. State nonuniform continuity criteria.
8. If $f : A \to \mathbb{R}$ is a Lipschitz function, then prove that f is uniformly continuous on A.
9. Prove that every constant function on [a, b] is in $\mathcal{R}[a, b]$.
10. If $f(x) = x^2$, for $x \in [0, 4]$, calculate the Riemann sum with respect to the partition $\dot{\mathcal{P}} = \{0, 1, 2, 4\}$ with tags at the left end points of the sub intervals.
11. Prove that the function $d(x, y) = |x - y|$ is a metric on $\mathbb{R}$.
12. Define closed set in a metric space. Give an example.
13. Investigate the convergence of $\int_0^1 \frac{1}{1-x}\, dx$.
14. Prove that $\int_1^\infty \frac{(1-e^{-x})}{x}\, dx$ diverges.

P.T.O.

15. Evaluate $\int_1^\infty \sqrt{x}e^{-x^2}dx$.

16. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Show that if f and g are uniformly continuous on $A \subseteq \mathbb{R}$ and if they are both bounded on A, then their product f g is uniformly continuous on A.

18. If $f \in \mathcal{R}[a, b]$, then prove that f is bounded.

19. Evaluate $\int_0^1 \frac{dx}{(x-1)^{\frac{2}{3}}}$.

20. Prove that $B(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$.

21. Prove that $\Gamma m\, \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}.\Gamma(2m)$.

22. Show that the sequence of functions $\left(\frac{x^n}{1+x^n}\right)$ does not converge uniformly on [0, 2].

23. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$. Then prove that f is continuous on A.

PART – D

Answer **any two** questions. **Each** question carries **6** marks.

24. State and prove continuous extension theorem.

25. Prove that a function $f \in \mathcal{R}[a, b]$ if and only if for every $\in > 0$ there exists $\eta_\in > 0$ such that if $\dot{\mathcal{P}}$ and $\dot{\mathcal{Q}}$ are any two tagged partitions of [a, b] with $\|\dot{\mathcal{P}}\| < \eta_\in$ and $\|\dot{\mathcal{Q}}\| < \eta_\in$, then $\lceil S(f, \dot{p}) - S(f, \dot{\mathcal{Q}}) \rceil < \in$.

26. Prove that if $f : [a, b] \to \mathbb{R}$ is monotone on [a, b], then $f \in \mathcal{R}[a, b]$.

27. State and prove Cauchy criterion for uniform convergence of sequence of functions.

K24U 0058

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – OBE – Regular/ Supplementary/Improvement) Examination, April 2024
(2019 to 2021 Admissions)
CORE COURSE IN MATHEMATICS
6B10 MAT : Real Analysis – II

Time : 3 Hours   Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Give an example of a step function defined on [1, 4].
2. Write norm of the partition P = (0, 5, 7, 9, 10) of [0, 10].
3. State additivity theorem.
4. Define Gamma function.
5. Define ε- neighborhood of a point x_0 in a metric space (S, d).

PART – B

Answer **any eight** questions. **Each** question carries **two** marks. **(8×2=16)**

6. State non-uniform continuity criteria for a function $f : A \to \mathbb{R}$.
7. Using an example, show that product of monotonic increasing functions need not be increasing.
8. Let $f(x) = x^2$, $x \in [0,5]$. Calculate Riemann sum with respect to the partition P = (0, 1, 3, 5), take tags at the left end point of the subintervals.
9. Show that value of the integral of a Riemann integrable function is unique.

10. If f is a Riemann integrable function and $k \in \mathbb{R}$, show that kf is Riemann integrable and $\int_a^b kf = k\int_a^b f$.

11. Evaluate $\int_1^\infty \frac{1}{x}dx$.

12. Show that B(m, n) = B(n, m).

13. Compute $\Gamma(-1/2)$.

14. Find pointwise limit of the sequence of functions (x^n) for $x \in [0,1]$.

15. Define a metric d on a set S.

16. State Cauchy criterion for convergence for sequence of functions.

PART – C

Answer **any four** questions. **Each** question carries **four** marks. **(4×4=16)**

17. Define uniformly continuous function. Show that $f(x) = x^2$ is not uniformly continuous on $[0, \infty)$.

18. Show that Riemann integrable functions defined on [a, b] are bounded on [a, b].

19. Show that if $f, g \in R[a,b]$, then $f + g \in R[a,b]$ and $\int_a^b (f+g) = \int_a^b f + \int_a^b g$.

20. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

21. From the definition of beta function, derive $B(m, n) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}} dy$.

22. Derive $\Gamma(n) = \int_0^\infty [\log(1/t)]^{n-1} dt$.

23. Show that a sequence of bounded functions (f_n) defined on a set A converges uniformly on A to a function f if and only if $\| f_n - f \| \to 0$.

PART – D

Answer **any two** questions. **Each** question carries **six** marks. **(6×2=12)**

24. a) Define a Lipschitz function. Show that Lipschitz functions are uniformly continuous.

    b) Show that not every uniformly continuous function is a Lipschitz function.

25. State and prove Fundamental theorem of calculus (1st form).

26. Show that $B(m,n) = \dfrac{\Gamma(m)\cdot\Gamma(n)}{(m+n)}$.

27. a) Using an example, show that pointwise limit of a sequence of continuous functions need not be continuous.

    b) Given that (f_n) is a sequence of continuous functions defined on a set A such that (f_n) converges uniformly to a function f defined on A. Prove that f is continuous on A.

K24U 0059

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – OBE – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2021 Admissions)
CORE COURSE IN MATHEMATICS
6B11 MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Define an analytic function.
2. Evaluate $\int_{-\pi i}^{\pi i} \cos z\,dz$.
3. Write Cauchy-Hadamard formula for radius of convergence.
4. Write Maclaurin's series expansion of $f(z) = e^z$.
5. State Picard's theorem.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks. **(8×2=16)**

6. Using the definition of derivative, show that $(z^2)' = 2z$.
7. Show that $\exp\left(\frac{\pi i}{2}\right) = i$.
8. Find In (1 + i)
9. Evaluate $\oint_C (z+1)^2\,dz$, where C is the unit circle.
10. Evaluate $\int_i^{i/2} e^{\pi z}\,dz$.
11. Evaluate $\int_0^1 (1+it)^2\,dt$.
12. Show that every power series $\sum_{n=0}^{\infty} a_n (z-z_0)^n$ converges at the center z_0.
13. State Taylor's theorem.

P.T.O.

14. Find center and radius of curvature of the power series $\sum_{n=0}^{\infty} \frac{(z-2i)^n}{n^n}$.

15. Find Laurent series expansion of $f(z) = \sin\frac{1}{z}$.

16. Define zero of a function. Give an example.

PART – C

Answer **any four** questions. **Each** question carries **four** marks. **(4×4=16)**

17. Use Cauchy-Riemann equations, show that e^z is an entire function.

18. Find an analytic function whose real part is $u(x, y) = x^2 + y^2$.

19. State and prove Cauchy's inequality.

20. Evaluate $\oint_C \frac{z^3-6}{(2z-i)^2}\,dz$, where C is the circle $|z| = 1$.

21. State and prove comparison test for convergence of a series $\sum_{n=1}^{\infty} zn$.

22. Explain different types of singular points with example.

23. Using residues, evaluate the integral $\oint_C \frac{e^{-z}}{z^2}dz$, where C is the circle $|z| = 3/2$.

PART – D

Answer **any two** questions. **Each** question carries **six** marks. **(2×6=12)**

24. Show that if $f(z) = u(x, y) + iv(x, y)$ is analytic in a domain D, then the partial derivatives of $u(x, y)$ and $v(x, y)$ satisfy Cauchy-Riemann equations.

25. State and prove Cauchy's integral formula.

26. a) Find the Maclaurin's series of $f(z) = \frac{1}{1+z^2}$.

    b) Find the Taylor series of $f(z) = \frac{1}{z}$ with center $z_0 = i$.

27. Give two Laurent series expansions with center at $z_0 = 0$ for the function $f(z) = \frac{1}{z^2(1-z)}$ and specify the region of convergence.

K23U 0514

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 and 2020 Admissions)
CORE COURSE IN MATHEMATICS
6B11 MAT : Complex Analysis

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　　　Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark :

1. Check whether $u = e^x \sin 2y$ is harmonic or not.
2. Evaluate $\int_{-\pi i}^{\pi i} \cos z \, dz$.
3. State Cauchy's integral theorem.
4. Discuss the convergence of $e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!}$.
5. Write the Maclaurin series for sinz.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks :

6. Find real part and imaginary part of $f(z) = \frac{1}{1-z}$ at $1-i$.
7. Check whether f(z) = cosx coshy – i sinx sinhy is analytic.
8. Define an entire function and write example of an entire function.
9. Evaluate $\int_C \operatorname{Re} z \, dz$, where C is the shortest path from 1 + i to 3 + 3i.

P.T.O.

10. Determine $\int_C \frac{1}{2z-1}\,dz$, where C is the unit circle in the counter clock wise direction.

11. Prove that if a series $z_1 + z_2 + \ldots\ldots..$ converges, then $\lim_{n\to\infty} z_n = 0$.

12. State root test for the convergence of a series.

13. Check the convergence of $\sum_{n=0}^{\infty} \frac{i^n}{n^2 - i}$.

14. State Laurent's theorem.

15. Evaluate $\oint_C \frac{1}{(z-1)(z-3)}\,dz, C : |z| = \frac{3}{2}$, in the counter clock wise direction.

16. Define zeros and singularities of a function f(z) and write example for each.

PART – C

Answer **any four** questions. **Each** question carries **four** marks :

17. Show that f(z) = $\bar{z}$ is nowhere differentiable.

18. Prove that $|\cos z|^2 = \cos^2 x + \sinh^2 y$.

19. State and prove Cauchy's integral formula.

20. State and prove Morera's theorem.

21. Define radius of convergence of a power series also find the radius of convergence of $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}(z-3i)^n$.

22. Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2 - 3z + 2}$ with center 0.

23. Find the residues at singular points of $\frac{\sin z}{z^3 - z}$.

PART – D

Answer **any two** questions. **Each** question carries **six** marks :

24. a) Find the value of z when $\ln z = 4 - 3i$.

    b) Express i^i in the form of a + ib.

    c) Write $e^{2+3\pi i}$ in the form of u + iv also find $|e^{2+3\pi i}|$.

25. Evaluate using Cauchy's integral formula.

    a) $\oint_C \frac{e^z}{z^n} dz$, where C is the unit circle in the counter clock wise direction.

    b) $\oint_C \frac{z+2}{z-2} dz, C : |z-1| = 2$, in the counter clock wise direction.

26. a) Find Maclaurin series for $f(z) = \sin(2z^2)$.

    b) Find Taylor series for $f(z) = \frac{1}{(z+i)^2}$ with center $z_0 = i$, also find radius of convergence.

27. a) State and prove Cauchy's residue theorem.

    b) Evaluate $\oint_C \frac{dz}{z^3(z-1)}, C : |z| = 2$, in the counter clock wise direction.

K23U 0515

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 and 2020 Admissions)
CORE COURSE IN MATHEMATICS
6B12MAT : Numerical Methods, Fourier Series and Partial Differential Equations

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark.

1. Forward difference operator $\Delta f(x_i)$ = __________
2. Using Picard's method, obtain a solution up to the second approximation to the equation $\frac{dy}{dx} = 2y - x$ such that y(0) = 1.
3. Define odd function and give an example.
4. Define a periodic function and find the period of $\cos \pi x$.
5. Write the Laplacian equation in Polar coordinates.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks.

6. Find the Lagrange interpolation polynomial for the following data :

x	1	2	4
f(x)	1	7	61

7. Find the second divided difference of $f(x) = \frac{1}{x}$, using points x_0, x_1, x_3.
8. Show that $\mu = \sqrt{\left(1 + \frac{1}{4}\delta^2\right)}$.

P.T.O.

9. Obtain the approximate value of y(1.2) for the initial value problem $y' = -2xy^2$, y(1) =1 using Taylor series second order with step size h = 0.1.

10. Using Euler method, find y(0.02), $y' = 2y$ with y(0) = 1 and h = 0.01.

11. Using Heun's method, find y(0.2), $y' = x^2 + y^2$ with y(0) = 1 and h = 0.1.

12. State Euler formula for Fourier coefficients.

13. Find the Fourier series of f(x) = x, – L < x < L, f(x + 2L) = f(x).

14. Verify that the function $u = x^2 + t^2$ is a solution of wave equation with suitable c.

15. Solve $u_{xx} - u = 0$.

16. Determine the type and normal form of the PDE $u_{xx} - 16u_{yy} = 0$.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Find ln(9.2) with n = 3, using Lagrange interpolation formula with the given table :

x	9.0	9.5	10	11.0
ln x	2.19722	2.25129	2.30259	2.39790

18. Using divided differences interpolation, find f(x) as a polynomial if

x	– 3	– 2	– 1	1	2	3
f(x)	18	12	8	6	8	12

19. Construct Newton's Backward Interpolation, table and Interpolating polynomial for the data :

x	– 4	– 2	0	2	4	6
f(x)	– 139	– 21	1	23	141	451

20. Using Picard's method, obtain a solution up to the fourth approximation to the equation $\frac{dy}{dx} = y + x$ such that y(0) = 1.

21. Given $\frac{dy}{dx} = 1 + y^2$ where y(0) = 0. Find y(0.2) correct to four decimal places by Runge-Kutta second order formula.

22. Find the Fourier series of the function f(x) = |x|, – 2 ≤ x ≤ 2 and f(x + 4) = f(x).

23. Consider the elastic string of length L whose ends are held fixed the string is set in motion from its equilibrium position with an initial velocity.

$$u_t(x, 0) = g(x) \begin{cases} \frac{2x}{L} & , \quad 0 \le x \le \frac{L}{2} \\ \frac{2(L-x)}{L}, & \frac{L}{2} \le x \le L \end{cases}$$

Find the displacement u(x, t) of the string.

PART – D

Answer **any two** questions. **Each** question carries **six** marks.

24. Using Lagrange interpolation, obtain the value of $e^{-0.15}$. Determine the maximum absolute error at this point. Compare it with actual error. If

x	0.1	0.2	0.4
f(x) = e^x	.904837	.818731	.670320

25. Use Runge-Kutta fourth-order method with h = 0.2 to find the value of y at x = 0.2 and x = 0.4, given $\frac{dy}{dx} = 1 + y^2$ where y(0) = 0.

26. Find the Fourier series of the function $f(x) = \begin{cases} x & , \quad \frac{-\pi}{2} < x \le \frac{\pi}{2} \\ (\pi - x), & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$.

27. Derive D'alembert solution of wave equation.

K23U 0229

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (C.B.C.S.S. – Supplementary) Examination, April 2023
(2017 to 2018 Admissions)
CORE COURSE IN MATHEMATICS
6B12MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. Find $(1 + i)^{16}$.
2. Determine the principal value of the argument of $-5 - 5i$.
3. State Taylor's theorem.
4. Develop a Maclaurin series of the function $\dfrac{1}{1-z^4}$.

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

5. Write $\dfrac{z_1 - z_2}{z_1 + z_2}$ of the form $x + iy$, where $z_1 = 4 + 3i$ and $z_2 = 2 - 5i$.
6. If $z = x + iy$, show that $\sin z = \sin x \cosh y + i \cos x \sinh y$.
7. Find the principal value of i^i.
8. Evaluate $\int_{8+\pi i}^{8-3\pi i} e^{\frac{z}{2}} dz$.
9. Integrate $\dfrac{z^2}{z^4 - 1}$ counter clockwise around the circle $|z + 1| = 1$.

P.T.O.

10. Integrate $f(z)=\dfrac{z^3+\sin z}{(z-i)^2}$ counter clockwise around the boundary of the square with vertices ± 2 and $\pm 2i$.

11. Find the radius of convergence of the power series $\sum_{n=0}^{\infty}\dfrac{n+5i}{(2n)!}(z-i)^n$.

12. Is the series $\sum_{n=1}^{\infty}\dfrac{(n!)^2}{(2n)!}$ convergent ? Justify your answer.

13. Determine the location and type of singularity of the function cot 2z.

14. Find $\operatorname{Res}_{z=i}\dfrac{9z+i}{z(z^2+1)}$.

SECTION – C

Answer **any four** questions. **Each** question carries **4** marks.

15. Verify triangle inequality for $z_1 = 4 - 6i$, $z_2 = 2 + 3i$.

16. If f(z) is analytic in a simply connected domain D, then show that the integral of f(z) is independent of path in D.

17. Show that $\int_C \dfrac{dz}{(z-z_1)(z-z_2)}=0$ for a simple closed path C enclosing z_1 and z_2.

18. State and prove root test for convergence of series.

19. Determine the location and order of the zero of $(z^4 - z^2 - 6)^3$.

20. Using Residue theorem, evaluate $\int_C \dfrac{z+1}{z^4-2z^3}\,dz$ where C is the circle $|z| = \dfrac{1}{2}$ (Counter clockwise).

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

21. Find all solutions of :
    a) $e^z = 1$
    b) $\cos z = 3i$.

22. State and prove M-L inequality. Using this show that $\left|\int_C \frac{dz}{z^4}\right| \le 4\sqrt{2}$ where C denote the line segment from z = i to z = 1.

23. a) Prove that a sequence $z_1, z_2, ..., z_n, ...$ of complex numbers $z_n = x_n + iy_n$ (where n = 1, 2, ...) converges to c = a + ib if and only if the sequence of real parts $x_1, x_2, ...$ converges to a and the sequence of imaginary parts $y_1, y_2, ...$ converges to b.

    b) Is the sequence $z_1, z_2, ..., z_n, ...$ where $z_n = \frac{n\pi i}{n+i}$ converges ? Justify.

24. Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2 - 3z + 2}$ with center 0.

K24U 0060

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2021 Admissions)
CORE COURSE IN MATHEMATICS
6B12 MAT : Numerical Methods, Fourier Series and Partial Differential Equations

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions out of **five** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Define an even function and give an example.
2. Define Newton's divided difference interpolation polynomial.
3. Perform 2 iterations of Picard's method to find an approximation solution of the initial value problem $y' = x + y^2$, $y(0) = 1$.
4. Find Half Range cosine series for $f(x) = x^2$ in $0 \leq x \leq \pi$.
5. Write Laplacian equation in polar coordinates.

PART – B

Answer **any eight** questions out of **eleven** questions. **Each** question carries **two** marks. **(8×2=16)**

6. Solve $u_{xy} = - u_x$.
7. Find the unique polynomial p(x) of degree 2 or less such that p(1) = 1, p(3) = 27 and p(4) = 64 using Lagrange interpolation formula.

P.T.O.

8. Write the normal form of the equation $AU_{xx} + 2BU_{xy} + CU_{yy} = F(x, y, U, U_x, U_y)$.

9. Prove that $\mu^2 = 1 + \frac{1}{4}\delta^2$.

10. Express $f(x) = \frac{1}{2}(\pi - x)$ as a Fourier series in the interval $0 \leq x \leq 2\pi$.

11. Determine the value of y when x = 0.1 given that y(0) = 1, $y' = x^2 + y, h = 0.05$.

12. Solve $\frac{dy}{dx} = 1 + xy$ with y(0) = 0 up to 3rd approximation by Picard's method of successive approximation.

13. Develop the Fourier series of $f(x) = x^2$ in $-2 \leq x \leq 2$.

14. Given $\frac{dy}{dx} = 1 + y^2$ where y = 0. When x = 0 find y(0.2).

15. Using the table find f as a polynomial in x,

x	– 1	0	3	6	7
f(x)	3	– 6	39	822	1611

16. Use Euler method to solve $\frac{dy}{dx} = x + xy + y, y(0) = 1$. Compute y at x = 0.15 by taking h = 0.15.

PART – C

Answer **any four** questions out of **seven** questions. **Each** question carries **four** marks. **(4×4=16)**

17. From the Taylor series for y(x) find y(0.1) correct to 4 decimal places if y(x) satisfies $y' = x - y^2$ and $y(0) = 1$.

18. Given the differential equation $\frac{dy}{dx} = \frac{x^2}{1 + y^2}$ with initial condition y = 0 when x = 0. Use Picard's method to obtain y for x = 0.25, 0.5 and 1.0, correct to 3 decimal places.

19. Using Lagrange's interpolation formula, find the form of the function y(x) from the following table :

x	0	1	3	4
y	– 12	0	12	24

20. Find the fourier series of the periodic function $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in the interval $(0, 2\pi)$.

21. Find the temperature u(x, t) in a laterally insulated copper bar 80 cm long. If the initial temperature is $100 \sin\left(\frac{\pi x}{80}\right)$ °C and the ends are kept at 0°C, how long will it take for the maximum temperature in the bar to drop to 50°C ?

    Physical data for copper : Density = 8.9 g/cm^3, Specific heat = 0.092 cal/g°C, thermal conductivity = 0.95 cal/cm sec.

22. Using Newton's forward difference formula, find the sum $s_n = 1^3 + 2^3 + 3^3 + ... + n^3$.

23. Values of x (in degrees) and sin x are given in the following table :

x (in degree)	sin x
15	0.2588190
20	0.3420201
25	0.4226183
30	0.5
35	0.5735764
40	0.6427876

Determine the value of sin 38°.

PART – D

Answer **any two** questions out of **four** questions. **Each** question carries **six** marks. **(2×6=12)**

24. Derive D'Alembert solution of wave equation.

25. A sinusoidal voltage E sin ωt where t is time, is passed through a half wave rectifier that clips the negative portion of the wave. Find the Fourier series of the resulting periodic function $u(t) = \begin{cases} 0 & \text{if } -L < t < 0 \\ E\sin\omega t & \text{if } 0 < t < L \end{cases}$

$p = 2L = \frac{2\pi}{\omega}, L = \frac{\pi}{\omega}$.

26. Using Runge-Kutta method of fourth order find y(0.2) from the initial value problem
$\frac{dy}{dx} = \frac{y - x}{y + x}, y(0) = 1$ taking h = 0.2.

27. From the following table values of x and y determine :
   i) f(0.23)
   ii) f(0.29)

x	**f(x)**
0.20	1.6596
0.22	1.6698
0.24	1.6804
0.26	1.6912
0.28	1.7024
0.30	1.7139

K24U 0394

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS – Supplementary/One Time Mercy Chance) Examination, April 2024
(2014 to 2018 Admissions)
Core Course in Mathematics
6B12MAT : COMPLEX ANALYSIS

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. If $z_1 = 8 + 3i$ and $z_2 = 9 - 2i$ then $\text{Im}(z_1 z_2) =$
2. Give an example for a function which has a simple pole at the point $z = 0$.
3. The residue of $f(z) = \dfrac{4}{1+z^2}$ at $z = i$ is
4. Define removable singularity. **(4×1=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Evaluate $\int_c \text{Re}(z)dz$, from $z = 0$ to $z = 1 + 2i$ along C, where C is the line segment joining the points (0, 0) and (1, 2).
6. Evaluate $\oint_C \dfrac{dz}{z-3i}$, where C is the circle $|z| = \pi$ in counter clockwise.
7. State and prove Liouville's theorem.
8. Define absolutely convergent and conditionally convergent of a series.

9. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(z-2i)^n}{n^n}$ and write its circle of convergence.

10. a) State ratio test.

 b) Prove that the derived series of a power series has the same radius of convergence as the original series.

11. Evaluate the residue of $\frac{9z+i}{z(z^2+1)}$ at z = i.

12. Find the Laurent series of $f(z) = z^2 e^{\frac{1}{z}}$ with center z = 0.

13. Define isolated essential singularity and pole of order m. Give an example for a function which has isolated essential singularity.

14. State Laurent's Theorem. **(8×2=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Verify that the $u(x, y) = x^3 - 3xy^2$ is harmonic in the whole complex plane and find a harmonic conjugate function v(x, y) of u(x, y).

16. a) Show that cosh z = coshx cosy + isinhx siny.

 b) Show that $|\cos z|^2 = \cos^2 x + \sinh^2 y$.

17. Expand $f(z) = \frac{1}{z(z-1)}$ in Laurent series valid for $0 < |z - 1| < 1$.

18. a) Give an example for a power series which is convergent only at its center.

 b) Prove that every power series $\sum_{n=1}^{\infty} a_n (z - z_0)^n$ converges at its center $z = z_0$.

 c) Prove that a power series $\sum_{n=1}^{\infty} a_n (z - z_0)^n$ converges at a point $z = z_1 \neq z_0$, is converges absolutely for every z closer to z_0 than z_1.

19. Evaluate $\oint_C \frac{e^{-z^2}}{\sin 4z} dz$, where C is the unit circle in counter clockwise.

20. Prove that if f(z) is analytic and has a pole at $z = z_0$ then $|f(z)| \to \infty$ as $z \to z_0$ in any manner. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. a) Show that the function $f(z) = 2x^2 + y + i(y^2 - x)$ satisfy the Cauchy – Riemann equation on the line y = 2x. Is it analytic on the line y = 2x ? Justify your answer.

    b) Prove that $\tanh^{-1} z = \frac{1}{2} \ln \frac{1+z}{1-z}$.

22. a) State and prove Cauchy – Riemann Equations.

    b) Find the principal value of $(2i)^{2i}$.

23. a) State and prove Cauchy's Integral formula.

    b) Evaluate $\oint_C \frac{z}{z^2 + 4z + 3} dz$, where C is the circle with center –1 and radius 2 in counter clockwise.

24. a) State and prove Cauchy's Inequality.

    b) Evaluate $\oint_C \frac{e^z}{(z-1)^2 (z^2+4)^2} dz$, for any contour C for which 1 lies inside and ±2i lie outside taken in counter clockwise. **(2×6=12)**

K23U 0230

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (C.B.C.S.S. – Supplementary) Examination, April 2023
(2017 to 2018 Admissions)
CORE COURSE IN MATHEMATICS
6B13MAT : Mathematical Analysis and Topology

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions, **each** question carries **1** mark.

1. If P = $\{a = x_0, x_1, x_2, \dots x_n = b\}$ is a partition of [a, b], then the Riemann lower sum of a function $f : [a, b] \to R$, is _________
2. Give an example of a sequence of continuous functions such that the limit function is not continuous.
3. A subset A of a topological space X is said to be dense if __________
4. Define the boundary point of a set A in a metric space X.

SECTION – B

Answer **any eight** questions, **each** question carries **2** marks.

5. If g(x) = x on [0, 1] and $P_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \dots \frac{n-1}{n}, 1\right\}$ then find $\lim\limits_{n\to\infty} (U(P_n, g) - L(P_n, g))$.
6. If f is continuous on [a, b], a < b, show that there exist $c \in [a, b]$ such that we have $\int_a^b f = f(c)(b - a)$.
7. Give an example for a bounded non-integrable function on [0, 1].
8. Define pointwise convergence and uniform convergence of a sequence of functions.

P.T.O.

9. If f_n is continuous on $D \subseteq R$ and if $\sum f_n$ converges to f uniformly on D, prove that f is continuous on D.

10. Determine the radius of convergence of the power series $\sum \frac{n^n}{n!} x^n$.

11. Let X be a non-empty set and define d by

$$d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$$

Show that d is a metric on X.

12. Prove that in a metric space X, each open sphere is an open set.

13. Prove that $\overline{A}$ equals the intersection of all closed supersets of A.

14. If T_1 and T_2 are 2 topologies on a non-empty set X, show that $T_1 \cap T_2$, is also a topology on X.

SECTION – C

Answer **any four** questions, **each** question carries **4** marks.

15. Show that if f : [a, b] $\rightarrow$ R is continuous on [a, b], then f is integrable on [a, b].

16. State and prove Darboux's theorem.

17. State and prove the Cauchy Criterion for Uniform Convergence.

18. Prove that every non-empty open set on the real line is the union of a countable disjoint class of open intervals.

19. Show that in a metric space X,

    a) any intersection of closed sets in X is closed.

    b) any finite union of closed sets in X is closed.

20. Show that a subset of a topological space is closed if and only if it contains its boundary.

SECTION – D

Answer **any two** questions, **each** question carries **6** marks.

21. If $f \in R[a, b]$ and if f is continuous at a point $c \in [a, b]$, prove that the indefinite integral $F(x) = \int_a^x f$ for $x \in [a, b]$ is differentiable at c and $F'(c) = f(c)$.

22. Prove that a sequence (f_n) of bounded functions on $A \subseteq R$ converges uniformly on A to f if and only if $\|f_n - f\|_A \to 0$.

23. State and prove Cantor's Intersection Theorem.

24. a) Let X and Y be topological spaces and f a mapping of X into Y. When do you say that f is :

    i) continuous

    ii) open

    iii) a homeomorphism ?

    b) Let X be a topological space, Y be a metric space, and A a subspace of X. If f is continuous mapping of A into Y, show that f can be extended in atmost one way to a continuous mapping of $\overline{A}$ into Y.

K23U 0516

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 and 2020 Admissions)
CORE COURSE IN MATHEMATICS
6B13 MAT : Linear Algebra

Time : 3 Hours | Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark.

1. Find the null space and range space of the zero transformation from R^3 to R^3.
2. Write a subspace of $M_{n \times n}$ (F).
3. What is the dimension of C over R ?
4. State Sylvester's law of nullity.
5. Give an example for an infinite dimensional vector space.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks.

6. Let T : $R^2 \to R^2$ defined by T(x, y) = (1, y). Is T linear ?
7. Prove that in any vector space V, 0x = 0, for each $x \in V$.
8. State Dimensional theorem.
9. Let T : $R^2 \to R^3$ defined by T(x, y) = (x + 7y, 2y). Write the matrix of T with respect to the standard ordered bases of R^2 and R^3.
10. If – 2 and 2 are eigen values of a square matrix A, then what are the eigen values of A′, transpose of A ?

P.T.O.

11. Let $T : F^2 \to F^2$ be a linear transformation defined by $T(x, y) = (1 + x, y)$. Find $N(T)$.

12. Determine whether $\{(2, -4, 1), (0, 3, -1), (6, 0, -1)\}$ form a basis for R^3.

13. Define an elementary matrix.

14. Let A be a 2×2 orthogonal matrix with 3 as an Eigen value. What will be the other Eigen value of A ?

15. Give an example for a linear transformation $T : F^2 \to F^2$ such that $N(T) = R(T)$.

16. State Cayley Hamilton theorem.

PART – C

Answer **any 4** questions. **Each** question carries **four** marks.

17. Define a vector space.

18. Prove that $P_n(F)$ is a vector space.

19. Prove that any intersection of subspaces of a vector space V is a subspace of V.

20. Prove that $\text{rank}(AA') = \text{rank}(A)$.

21. Find the rank of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{bmatrix}$.

22. Let W be a subspace of a finite dimensional vector space V. Then prove that W is finite dimensional and $\dim W \le \dim V$. Moreover if $\dim W = \dim V$ then prove that $V = W$.

23. Let $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. Find A^{-1} using Cayley Hamilton theorem.

PART – D

Answer **any 2** questions. **Each** question carries **six** marks.

24. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ into normal form and hence find the rank.

25. Solve the system of equations

    $x + 3y - 2z = 0$, $2x - y + 4z = 0$, $x - 11y + 14z = 0$.

26. Find the Eigen values and Eigen vectors of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

27. State and prove replacement theorem.

K24U 0061

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2021 Admissions)
CORE COURSE IN MATHEMATICS
6B13 MAT : Linear Algebra

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark.

1. Define subspace of a vector space.
2. What is the dimension of the vector space of all 2×3 matrices over R ?
3. State Dimension Theorem.
4. The characteristic roots of a matrix A are 2, 3 and 4. Then find the characteristic roots of the matrix 3A.
5. Find the eigen values of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks.

6. Let $V = \{(a_1, a_2) : a_1, a_2 \in R\}$. Define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, 0)$ and $c(a_1, a_2) = (ca_1, 0)$. Is V a vector space over R with these operations ? Justify your answer.
7. Prove that the set of all symmetric matrices of order n is a subspace of the vector space of all square matrices of order n.

P.T.O.

8. Check whether the set {(1, –1, 2), (2, 0, 1), (–1, 2, –1)} is linearly independent or not.

9. Give an example of three linearly dependent vectors in R^3 such that none of the three is a multiple of another.

10. Find the rank of matrix A, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$.

11. Show that rank of a matrix, every element of which is unity, is 1.

12. Show that $T : R^2 \to R^2$ defined by $T(a_1, a_2) = (a_1 + a_2, a_1)$ is a linear transformation.

13. Explain the condition for consistency and nature of solution of a non homogeneous linear system of equations AX = B.

14. Let $T : V \to V$ be a linear transformation. Find the range and null space of zero transformation and identity transformation.

15. Prove that the Eigen values of an idempotent matrix are either zero or unity.

16. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$.

PART – C

Answer **any 4** questions. **Each** question carries **four** marks.

17. Prove that any intersection of subspaces of a vector space V is a subspace of V.

18. Suppose that $T : R^2 \to R^2$ is linear, T(1,0) = (1,4) and T(1,1) = (2,5). What is T(2,3) ? Is T one-to-one ?

19. Let $T : R^2 \to R^3$ be defined by $T(a_1, a_2) = (a_1 - a_2, a_1, 2a_1 + a_2)$. Let β be the standard ordered basis for R^2 and γ = { (1, 1, 0), (0, 1, 1), (2, 2, 3)}. Compute $[T]_\beta^\gamma$.

20. Under what condition the rank of the following matrix A is 3 ? Is it possible for the rank to be 1 ? Why ? $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & 1 & 2 \\ 1 & 0 & x \end{bmatrix}$.

21. Solve the system of equations.

$x - 2y + 3z = 0$

$2x + y + 3z = 0$

$3x + 2y + z = 0$

22. Find the eigen vectors of the matrix $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

23. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ find A^2 using Cayely Hamilton theorem and then find A^3.

PART – D

Answer **any 2** questions. **Each** question carries **six** marks.

24. Prove that the set of all $m \times n$ matrices with entries from a field F is a vector space over F with the operations of matrix addition and scalar multiplication.

25. Find the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary row operations.

26. Find the values of a and b for which the system of equations

$x + y + z = 3$

$x + 2y + 2z = 6$

$x + 9y + az = b$ have

1) no solution;
2) unique solution and;
3) an infinite number of solutions.

27. Using Cayley Hamilton theorem find the inverse of $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.

K24U 0395

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – Supplementary/One Time Mercy Chance) Examination, April 2024
(2014 to 2018 Admissions)
Core Course in Mathematics
6B13MAT : MATHEMATICAL ANALYSIS AND TOPOLOGY

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. If $P = \{a = x_0, x_1, x_2, \ldots x_n = b\}$ is a partition of [a, b], then the Riemann upper sum of a function $f : [a, b] \to R$, is
2. Evaluate $\lim(f_n(x))$ where $f(x) = \dfrac{nx}{(1+n^2x^2)}$ for $x \in R$, $n \in N$.
3. A topological space is said to be separable if it has
4. Let X be an arbitrary metric space and $A \subseteq X$. Then Int(A) = **(4×1=4)**

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

5. If $h(x) = x^2$ on [0, 1] and $P_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \ldots \frac{n-1}{n}, 1\right\}$ then find $\lim\limits_{n\to\infty} (U(P_n, h) - L(P_n, h))$.
6. If $f \in R\,[a, b]$ and $|f(x)| \leq M$ for all $x \in [a, b]$, then show that $\left|\int_a^b f\right| \leq M(b-a)$.
7. Give an example for a bounded nonintegrable function on [0, 1].
8. Discuss the convergence of sequence (x^n) for $x \in R$.
9. State Weierstrass M-Test.
10. Determine the radius of convergence of the power series $\sum \left(1+\frac{1}{n}\right)^{n^2} x^n$.
11. Let X be a non-empty set and define d by $d(x, y) = \begin{cases} 0 & \text{if } x = y \\ 1 & \text{if } x \neq y \end{cases}$. Show that d is a metric on X.

P.T.O.

12. Prove that in a metric space X, the complement of a closed set is open.

13. Prove that $\overline{A}$ equals the intersection of all closed supersets of A.

14. Show that the intersection of two topologies on a non-empty set X is also a topology on X. **(8×2=16)**

SECTION – C

Answer **any four** questions. **Each** question carries **4** marks.

15. Show that if $f : [a, b] \to R$ is monotone on [a, b], then f is integrable on [a, b].

16. State and prove the Fundamental Theorem of Calculus (First Form).

17. If $\{f_n\}$ is a sequence of continuous functions on a set $A \subseteq R$ converging uniformly on A to a function $f : A \to R$, then f is continuous on A.

18. Show that a subset of a topological space is perfect if and only if it is closed and has no isolated points.

19. If X is a complete metric space and Y is a subspace of X, prove that Y is complete if and only if it is closed.

20. Let X be an infinite set. Show that
$T = \{U \subseteq X : U = \phi \text{ or } X \setminus U \text{ is finite}\}$ is a topology on X. **(4×4=16)**

SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

21. State and prove Riemann's Criterion for integrability.

22. Let (f_n) be a sequence of bounded functions on $A \subseteq R$. Prove that this sequence converges uniformly on A to a bounded function f if and only if for each $\in > 0$ there is number $H(\in)$ in N such that for all $m, n \geq H(\in)$, then $||f_m - f_n|| A \leq \in$.

23. Show that in a metric space X :
    a) any union of open sets is open and
    b) any finite intersection of open sets is open.

24. Let $f : X \to Y$ be a mapping of one topological space into another. Show that f is continuous if and only if $f^{-1}(F)$ is closed in X whenever F is closed in Y. **(2×6=12)**

K23U 0517

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 and 2020 Admissions)
DISCIPLINE SPECIFIC ELECTIVE IN MATHEMATICS
6B14A MAT : Graph Theory

Time : 3 Hours Max. Marks : 48

PART – A

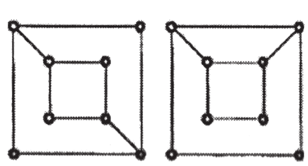
Answer **any 4** questions. **Each** question carries **one** mark.

1. Define Graph.
2. Define connectivity of a graph.
3. Draw a 3-regular graph.
4. Define Euler tour.
5. What is meant by adjacency matrix of a graph ?

PART – B

Answer **any 8** questions. **Each** question carries **two** marks.

6. Define union and intersection of sub graphs of a graph.
7. Are the following graphs isomorphic ? Justify your answer.



P.T.O.

8. Find the number of vertices in a complete graph with 55 edges.

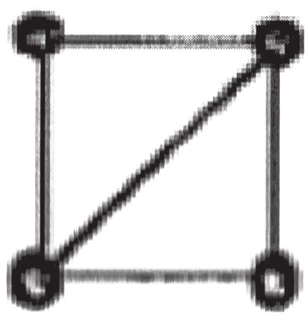
9. Draw all trees with 5 vertices.

10. Define platonic bodies.

11. Define walk. Give one example.

12. Explain Chinese Postman Problem.

13. State Euler's formula. Verify the formula in the following graph.



14. What is meant by closure of a graph ?

15. Draw a complete bipartite non planar graph.

16. Find the number of distinct spanning trees in the complete graph K_5.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. State **True** or **False**. Graphs are natural mathematical models. Justify your answer.

18. Prove that a connected graph is a tree if and only if every edge of G is a bridge.

19. Prove that a simple graph G is Hamiltonian if and only if its closure C(G) is Hamiltonian.

20. Prove that a connected graph G with at most two odd vertices has an Euler trail.

21. Let G be a graph with n vertices. Prove that if G is a connected graph with $n - 1$ edges then G is a tree.

22. a) Define Jordan curve. Give one example.

    b) State Jordan curve theorem.

23. Explain contraction with example.

PART – D

Answer **any two** questions. **Each** question carries **six** marks.

24. Prove that a tree with n vertices has precisely $n - 1$ edges.

25. a) State and prove the first theorem of graph theory.

    b) Prove that every graph has an even number of odd vertices.

    c) Let G be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.

26. Prove that a connected graph is Euler iff the degree of every vertex is even.

27. Prove that k_5, the complete graph on five vertices, is non planar.

K24U 0062

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (C.B.C.S.S. – OBE – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2021 Admissions)
DISCIPLINE SPECIFIC ELECTIVE IN MATHEMATICS
6B14A MAT : Graph Theory

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark.

1. Define a simple graph.
2. Define a vertex deleted subgraph.
3. Define the adjacency matrix of a graph.
4. Define the vertex connectivity of a graph.
5. State Kuratowski's theorem.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks.

6. Draw all non isomorphic simple graphs with 3 vertices.
7. By considering two graphs G_1 and G_2 on three vertices, draw $G_1 \cap G_2$ and $G_1 \cup G_2$.
8. Define a self complementary graph. Draw a graph which is self complementary.
9. For a connected graph G, define the terms diameter and eccentricity.
10. Define a tree and sketch two isomorphic trees on 4 vertices.
11. Draw Petersen graph and determine the vertex connectivity of the Petersen graph.
12. Define a tour and an Euler tour of a graph G.

P.T.O.

13. Define Hamiltonian graph. Draw a graph with Hamiltonian path but no Hamiltonian cycle.
14. Explain the travelling salesman problem.
15. State Jordan curve theorem and give an example of a complete graph which is nonplanar.
16. Verify Euler's formula for wheel graph W_4.

PART – C

Answer **any 4** questions. **Each** question carries **four** marks.

17. State and prove the first theorem of graph theory.
18. Let G be an acyclic graph with n vertices and k connected components. Then prove that G has n-k edges.
19. Prove that a connected graph with n vertices and n-1 edges is a tree.
20. Define closure of a graph with example.
21. Prove that a simple graph G is Hamiltonian if and only if its closure C(G) is Hamiltonian.
22. Show that $K_{3,3}$ is nonplanar.
23. Explain contraction with example.

PART – D

Answer **any 2** questions. **Each** question carries **six** marks.

24. Define a complete graph and complete bipartite graph. Give an example of a complete bipartite graph which is complete. Also sketch the complete graphs with at most 6 vertices.
25. Prove that a graph G is connected if and only if it has a spanning tree.
26. Prove that a connected graph G is Euler if and only if the degree of every vertex is even.
27. Let G be a simple 3 connected graph with at least 5 vertices. Then prove that G has a contractible edge.