K23U 2365

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2023
(2019-2021 Admissions)
CORE COURSE IN MATHEMATICS
5B05 MAT : Set Theory, Theory of Equations and Complex Numbers

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions from this part. **Each** question carries **1** mark. **(4×1=4)**

1. Give example for a denumerable set.
2. If α, β, γ are the root of the equation f(x) = 0, then the equation whose roots are $-\alpha, -\beta, -\gamma$ is ______
3. Show that $x^5 - 2x^2 + 7 = 0$ has atleast two imaginary roots.
4. If ω is an imaginary cube root of unity, then the value of $1 + \omega + \omega^2$ is ______
5. What is the value of Arg z for positive real axis, z = x ?

PART – B

Answer **any 8** questions from this part. **Each** question carries **2** marks. **(8×2=16)**

6. Show that the set of all integers is countable.
7. If α, β, γ are the root of the equation $ax^3 + bx^2 + cx + d = 0$, then find the values of $\alpha + \beta + \gamma$ and $\alpha\beta\gamma$.
8. Find the condition that the cubic equation $x^3 - lx^2 + mx - n = 0$ should have its roots in arithmetical progression.
9. If α, β, γ are the root of the equation $8x^3 - 4x^2 + 6x - 1 = 0$, find the equation whose roots are $2\alpha + 1$, $2\beta + 1$, $2\gamma + 1$.
10. State De Gua's rule.
11. What do you mean by reciprocal equation ? Give an example.

P.T.O.

12. Describe the discriminant of the cubic equation $ax^3 + 3bx^2 + 3cx + d = 0$.

13. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking the second term.

14. If a, b, c are the roots of the cubic equation $x^3 + px^2 + qx + r = 0$, find the value of $\frac{1}{a^2b^2} + \frac{1}{b^2c^2} + \frac{1}{c^2a^2}$.

15. What are the imaginary cube root of unity ?

16. Find the polar form of $z = 1 + i$.

PART – C

Answer **any 4** questions from this part. **Each** question carries **4** marks. **(4×4=16)**

17. If A is a set with m elements and B is a set with n elements and if $A \cap B = \phi$, then prove that $A \cup B$ has m + n elements.

18. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$, given that the sum of the two of its roots is zero.

19. Find the rational roots of $x^4 - 39x^2 + 46x - 168 = 0$.

20. Solve $6x^5 + 11x^4 - 33x^2 + 11x + 6 = 0$.

21. Describe the behaviour of roots of a cubic equation in terms of its discriminant.

22. Find the value of $\sqrt{1+i}$.

23. Find the fifth root of (–1).

PART – D

Answer **any 2** questions from this part. **Each** question carries **6** marks. **(2×6=12)**

24. State and prove Cantor's theorem.

25. If α, β, γ are the root of the equation $ax^3 + 3bx^2 + 3cx + d = 0$, then find the values of

    a) $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$

    b) $(\beta - \gamma)(\gamma - \alpha) + (\gamma - \alpha)(\alpha - \beta) + (\alpha - \beta)(\beta - \gamma)$.

26. Find a real root of the $x^3 + x^2 - 16x + 20 = 0$.

27. If z_1 and z_2 are two complex numbers, prove that

    a) $|z_1z_2| = |z_1|\,|z_2|$

    b) $\arg(z_1z_2) = \arg z_1 + \arg z_2$.

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V Semester B.Sc. Degree (C.B.C.S.S.-O.B.E.-Regular/Supplementary/ Improvement) Examination, November 2023
(2019 – 2021 Admissions)
CORE COURSE IN MATHEMATICS
5B06 MAT : Real Analysis – I

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. They carry **1** mark **each**. **(4×1=4)**

1. State Triangle Inequality.
2. Find $\lim\left(1+\frac{1}{2n}\right)^n$.
3. Define m-tail of a sequence.
4. Define continuity of a function at a point.
5. Define Rearrangement of the series.

PART – B

Answer **any 8** questions from among questions **6** to **16**. These questions carry **2** marks **each**. **(8×2=16)**

6. Determine the set A of $x \in R$ such that $|2x + 3| < 8$.
7. If $a \in R$ and $a \neq 0$ then show that $a^2 > 0$.
8. Discuss the convergence of $\lim\left(\frac{n}{2^n}\right)$.
9. Find the limit of the sequence whose terms are given by $x_1 = 8$, $x_{n+1} = \frac{x_n}{2} + 2$ for $n \in N$.
10. State Monotone Convergence Theorem.
11. Define subsequence of a sequence with an example.
12. State Alternating Series test.

P.T.O.

13. Define convergent Series.
14. If Σa_n with $a_n > 0$ is convergent, then is $\sum \sqrt{a_n}$ always convergent. Justify.
15. Show that $f(x) = \frac{1}{x}$ defined on A = (0, ∞) is unbounded on A.
16. State Boundedness Theorem.

PART – C

Answer **any 4** questions from among questions **17** to **23**. These questions carry **4** marks **each**. **(4×4=16)**

17. Show that cosine function is continuous on R.
18. Discuss the convergence of $\sum_{n=0}^{\infty} r^n$, $r \in R$, $|r| < 1$.
19. Discuss the convergence of $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.
20. Discuss the convergence of the sequences
    a) $((-1)^n)$ and
    b) (n).
21. Show that Cauchy sequence of real numbers is bounded.
22. State and prove Archimedean property.
23. If a and b are positive real numbers, a ≠ b then show that $\sqrt{ab} \le \frac{(a+b)}{2}$.

PART – D

Answer **any 2** questions from among questions **24** to **27**. These questions carry **6** marks **each**. **(2×6=12)**

24. State and prove density theorem of rational numbers in R.
25. State and prove Squeeze theorem for sequences. Hence find $\lim\left(\frac{\sin n}{n}\right)$.
26. Discuss the convergence of
    a) $\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)}$
    b) $\sum_{n=1}^{\infty} \frac{(\cos n)}{n^2}$.
27. Discuss the continuity of
    a) Dirichlet's function
    b) Thomae's function.

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V Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2023
(2019-2021 Admissions)
CORE COURSE IN MATHEMATICS
5B07 MAT : Abstract Algebra

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark : **(4×1=4)**

1. Give an example of a finite group that is not cyclic.

2. Find the order of the element 4 in Z_6.

3. What is the order of the permutation (124) (23) in S_6 ?

4. Define Kernel of a homomorphism.

5. Find all solutions of the equation $x^2 + 2x + 2 = 0$ in Z_6.

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks : **(8×2=16)**

6. Find the group table of the Klein 4-group. List all its subgroups.

7. Show that every cyclic group is abelian. Discuss its converse.

8. Let S be the set of all real numbers except – 1. Define $*$ on S by $a + b = a + b + ab$. Check whether $(S,*)$ is a group or not.

9. Find all the generators of Z_{18}.

P.T.O.

10. Find the number of elements in the set $\{\sigma \in S_5 | \sigma(2) = 5\}$.

11. Define odd permutation. Give an example of an odd permutation in S_4.

12. Prove that a group homomorphism ϕ defined on G is one-to-one if and only if $\ker(\phi) = \{e\}$.

13. Consider $\gamma : Z \to Z_n$ by $\gamma(m) = r$, where r is the remainder when m divided by n. Show that γ is a group homomorphism. What is its kernel ?

14. Show that the cancellation law with respect to multiplication hold in a ring R if and only if R has no divisors of zero.

15. Show that every field is an integral domain. Discuss its converse.

16. Define characteristic of a ring. What is the characteristic of the ring Z_6 ?

PART – C

Answer **any 4** questions from this Part. **Each** question carries **4** marks : **(4×4=16)**

17. Let G be a group and let a be one fixed element of G. Show that the set $H_a = \{x \in G | xa = ax\}$ is a subgroup of G.

18. Show that every permutation of a finite set can be written as a product of disjoint cycles.

19. Let G be a group of order pq, where p and q are prime numbers. Show that every proper subgroup of Z_{pq} is cyclic.

20. Let H be a subgroup of a group G such that $ghg^{-1} \in H$ for all $g \in G$ and all $h \in H$. Show that $gH = Hg$.

21. Let $\phi : G \to G'$ be a group homomorphism with kernel H and let $a \in G$. Show that $\{x \in G | \phi(x) = \phi(a)\} = aH$.

22. Show that the map $\phi : Z \to Z_n$ where $\phi(a)$ is the remainder of a modulo n is a ring homomorphism.

23. An element a of a ring R is idempotent of $a^2 = a$. Show that a division ring contains exactly two idempotent elements.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks : **(2×6=12)**

24. State and prove Cayley's theorem.

25. Let H be a subgroup of a group G. Then show that the left coset multiplication (aH) (bH) = abH is well-defined if and only if H is a normal subgroup of G.

26. Show that if a finite group G has exactly one subgroup H of a given order, then H is a normal subgroup of G.

27. Show that the characteristic of an integral domain must be 0 or a prime number. Give examples of two non-isomorphic rings with characteristic 4.

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V Semester B.Sc. Degree (CBCSS – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2023
(2019 – 2021 Admissions)
CORE COURSE IN MATHEMATICS
5B08 MAT : Differential Equations and Laplace Transforms

Time : 3 Hours Max. Marks : 48

PART – A

(Short Answer)

Answer **any four** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Solve the differential equation $y' = 1 + y^2$.

2. Check whether the equation $-ydx + xdy = 0$ is exact.

3. Give an example of a non-homogeneous differential equation.

4. Solve $y'' - y = 0$.

5. State the linearity property of the Laplace transform.

PART – B

(Short Essay)

Answer **any eight** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Find the order and degree of the differential equation $\frac{d^2y}{dx^2} + \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}} = 0$.

7. Prove that e^x is an integrating factor of siny dx + cosy dy = 0 and solve it.

P.T.O.

8. Find the orthogonal trajectories of the curve $y = ce^{-x}$.

9. State the existence theorem of first order differential equations.

10. Solve the initial value problem $y'' - y' - 2y = 0$, $y(0) = -4$, $y'(0) = -17$.

11. Check whether the solutions x^2 and $x^2 \ln x$ are linearly independent.

12. Find the Laplace transform of $a + bt + ct^2$.

13. Solve $y'' + 25y = 0$.

14. Find the inverse Laplace transform of $\frac{12}{(s-3)^4}$.

15. Write the standard form of Euler Cauchy equation. Give an example.

16. Solve $2x \tan y \, dx + \sec^2 y \, dy = 0$.

PART – C

(Essay)

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

17. Find the general solution of $y' - y = e^{2x}$.

18. Solve $y'' + 2y' + y = x^2$.

19. Let $f(t) = t \sin wt$, find the Laplace transform of $f(t)$.

20. Check for exactness and solve the initial value problem, $ye^x dx + (2y + e^x)dy = 0$, $y(0) = -1$.

21. Solve $y' = (y + 4x)^2$.

22. Solve $(\cot y + x^2)dx = x \csc^2 y \, dy$.

23. Solve $y'' + y = \sec x$.

PART – D
(Long Essay)

Answer **any two** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

24. Using Laplace transforms, solve the integral equation, $y(t) = 1 - \int_0^t (t - \tau)\, y(\tau) d\tau$.

25. Solve $4x^2D^2 + 24xD + 25\, y = 0$, $y(1) = 2$, $y'(1) = -6$.

26. Solve $y'' + 2y' + 5y = 1.25e^{0.5x} + 40 \cos 4x - 55 \sin 4x$, $y(0) = 0.2$, $y'(0) = 60.1$.

27. Find an integrating factor and solve the initial value problem

$$2\sin(y^2)dx + xy\cos(y^2)dy = 0,\ y(2) = \sqrt{\frac{\pi}{2}}.$$

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Reg. No. : ....

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V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2023
(2019 – 2021 Admissions)
CORE COURSE IN MATHEMATICS
5B09MAT: Vector Calculus

Time : 3 Hours Max. Marks : 48

PART – A

(Short Answer Questions)

Answer **any four** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Find the parametric equation for the line through (3, –4, –1) parallel to the vector v = i + j + k.

2. Find the distance from the point (2, –3, 4) to the plane x + 2y + 2z = 13.

3. Find the gradient of the function $f(x, y) = xy^2$ at the point (2, –1).

4. Evaluate $\int_C (x + y)ds$, where C is the straight line segment x = t, y = 1 – t, z = 0 from (0, 1, 0) to (1, 0, 0).

5. Define Divergence Theorem.

PART – B

(Short Essay Questions)

Answer **any eight** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Find the length of the portion of the curve r(t) = 4cost i + 4sint j + 3t k, $0 \le t \le \frac{\pi}{2}$.

7. Find the curvature of r(t) = 3sint i + 3cost j + 4t k.

P.T.O.

8. Find the directions in which $f(x,y)=\frac{x^2}{2}+\frac{y^2}{2}$ increases more rapidly at (1, 1).

9. Find the plane tangent to the surface $z = x\cos y - ye^x$ at (0, 0, 0).

10. Find the work done by the force field F = xi + yj + zk in moving an object along the curve C parametrized by $r(t) = \cos(\pi t)i + t^2 j + \sin(\pi t)k$, $0 \le t \le 1$.

11. Find the scalar potential of the vector field F = 2xi + 3yj + 4zk.

12. Find the Curl of $F = (x^2 - z)i + xe^z j + xyk$.

13. Find the critical points of the function $f(x, y) = x^2 + y^2 - 4y + 9$.

14. Find the Divergence of the vector field $F = (y^2 - x^2)i + (x^2 + y^2)j$.

15. Integrate $G(x, y, z) = x^2$ over the cone $z=\sqrt{x^2+y^2}, 0 \le z \le 1$.

16. Evaluate $\int_C y^2dx + x^2dy$, $C : x^2 + y^2 = 4$.

PART – C

(Essay Questions)

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

17. Find the angle between the planes 2x + 2y + 2z = 3, 2x – 2y – z = 5.

18. Find the unit tangent vector of the curve
$r(t) = \sin t\, i + (3t^2 - \cos t)j + e^t k$, at $t_0 = 0$.

19. Find the derivative of $f(x, y, z) = x^3 - xy^2 - z$ at (1, 1, 0) in the direction of v = 2i – 3j + 6k.

20. Verify Green's theorem for F = –yi + xj over the circle
C : acost i + asint j, $0 \le t \le 2\pi$.

21. Verify Divergence theorem for F = xi + yj + zk over the sphere $x^2 + y^2 + z^2 = a^2$.

22. Find the linearization L(x, y, z) of f(x, y, z) = $x^2 - xy + 3\sin z$ at the point (2, 1, 0).

23. Integrate G (x, y, z) = xyz over the surface of the cube cut from the first octant by the planes x = 1, y = 1, z = 1.

PART – D

(Long Essay Questions)

Answer **any two** questions from this Part. **Each** question carries **6** marks. **(2×6=12)**

24. Find the curvature and torsion of the curve

    r(t) = (cost + tsint)i + (sint –tcost)j, t > 0.

25. Find the local extreme values of the function f(x, y) = $xy - x^2 - y^2 - 2x - 2y + 4$.

26. Show that ydx + xdy + 4dz is exact and evaluate the integral

    $\int$ ydx + xdy + 4dz over any path from (1, 1, 1) to (2, 3, –1).

27. Find the center of mass of a thin hemispherical shell of radius a and constant density δ.