K23U 1127

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 Admission Onwards)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
4C04 MAT-PH : Mathematics for Physics – IV

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark.

1. Define the gradient field of a differentiable function f(x, y, z).

2. Define the circulation density of a vector field F = Mi + Nj at the point (x, y).

3. Give an example for a non-orientable surface.

4. State the Trapezoidal rule for Numerical Integration.

5. Find the order of the partial differential equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial u}{\partial x}$. **(4×1=4)**

PART – B

Answer **any seven** questions. **Each** question carries **2** marks.

6. Find the curl of $F = (x^2 - z)i + xe^z j + xyk$.

7. State Stoke's theorem for a smooth oriented surface.

8. With the usual notations, prove that $\nabla \times \nabla f = 0$.

9. The vector field F(x, y, z) = xi + yj + zk represent the velocity of a gas flowing in space. Show that the gas is undergoing constant uniform expansion at all points.

10. Find a parametrization of the cone $z = \sqrt{x^2 + y^2}$, $0 \le z \le 1$.

11. Evaluate the line integral $\int_C xydy - y^2dx$ where C is the square cut from the first quadrant by the lines x = 1 and y = 1.

12. Evaluate $\int_{-3}^{3} x^4\,dx$ by using Simpson's 1/3 rule.

13. Evaluate $\int_0^6 \frac{1}{1+x}dx$ using Trapezoidal rule.

14. Describe the fourth order Runge-Kutta formula.

15. Show that $u = x^2 - y^2$ is a solution of the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

16. Solve $u_{xy} = -u_x$. **(7×2=14)**

PART – C

Answer **any four** questions. **Each** question carries **3** marks.

17. Find the work done by the conservative field F = yzi + xzj + xyk along any smooth curve C joining the point A(– 1, 3, 9) to B(1, 6, – 4).

18. Find the flux of F = (x – y)i + xj across the circle $x^2 + y^2 = 1$ in the xy-plane.

19. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to the point (1, 1, 1).

20. Apply Simpson's one third rule to evaluate $\int_1^6 \frac{1}{1+x^2}dx$ with h = 1.

21. From the Taylor series for y(x), find y(0.1) correct to four decimal places if y(x) satisfies $y' = x - y^2$, y(0) = 1.

22. Solve the wave equation $u_{tt} - c^2u_{xx} = 0$.

23. If u_1 and u_2 are solutions of $u_t = c^2u_{xx}$ in some region R. Prove that $u = c_1u_1 + c_2u_2$ is also a solution of the above partial differential equation.

(4×3=12)

PART – D

Answer **any two** questions. **Each** question carries **5** marks.

24. Find a parametrization of the cylinder $x^2 + (y - 3)^2 = 9$, $0 \leq z \leq 5$.

25. Integrate $G(x, y, z) = xyz$ over the surface of the cube cut from the first octant by the planes $x = 1$, $y = 1$ and $z = 1$.

26. Given : $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$. Find $y(0.2)$ and $y(0.4)$.

27. Find the type, transform to normal form, and solve the partial differential equation $u_{xx} + 5u_{xy} + 6u_{yy} = 0$. **(2×5=10)**

K23U 1128

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 Admission Onwards)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
4C04 MAT-CH : Mathematics for Chemistry – IV

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions. **Each** question carries **1** mark.

1. Write an example of a one dimensional heat equation.
2. Write the solution of one dimensional heat equation with the binary condition $u(0, t) = u(L, t) = 0 \ \forall \ t \geq 0$.
3. Write Simpson's 1/3-rule of integration.
4. Give an example of a group.
5. Define order of a group. **(4×1=4)**

PART – B

Answer **any seven** questions. **Each** question carries **2** marks.

6. Solve $u_{xx} = 0$.
7. For what values of c, the function $u(x, t) = x^2 + t^2$ satisfies the wave equation ?
8. Identify the type of the partial differential equation $u_{xx} - 16u_{yy} = 0$.
9. Write the condition that the PDE

   $Au_{xx} + 2Bu_{xy} + Cu_{yy} = F(x, y, u, u_x, u_y)$ is Elliptic and parabolic.
10. Solve $25u_{yy} - 4u = 0$.

P.T.O.

11. Evaluate $\int_1^3 \frac{1}{x} dx$ using trapezoidal rule with n = 5.

12. Write Euler's method and modified Euler's methods for solving differential equations.

13. Write the Taylor series expansion of a function at $x = x_0$.

14. Give an example of an abelian and non abelian groups.

15. Define cyclic groups. Give an example. **(7×2=14)**

PART – C

Answer **any four** questions. **Each** question carries **3** marks.

16. Write any three physical assumptions for deriving wave equation.

17. By using the method of characteristic, solve $u_{xx} + 4u_{yy} = 0$.

18. Find y (0.1) for the differential equation for
$y'(x) = x - y^2$, y (0) = 1.

19. Evaluate $\int_0^1 \frac{1}{1+x^2} dx$, using trapezoidal rule with h = 0.5.

20. Give the multiplication table of a group of order 3.

21. Describe the rotation of a molecule.

22. Write any three applications of group theory in chemistry. **(4×3=12)**

PART – D

Answer **any two** questions. **Each** question carries **5** marks.

23. Derive the D'Alembert's solution of $\frac{\partial^2 u}{\partial t^2} = C^2 \frac{\partial^2 u}{\partial x^2}$.

24. Evaluate $\int_0^1 \sqrt{1-x^2}$ using with n = 5 numerically.

25. Determine the value of y when x = 0.1 given that y (0) = 1, $y' = x^2 + y$, with h = 0.05.

26. State and prove rearrangement theorem of group multiplication table. **(2×5=10)**

K24U 0732

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2022 Admissions)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
4C04 MAT-CH : Mathematics for Chemistry – IV

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions. **Each** question carries **1** mark.

1. Write the standard form of a Two-dimensional Laplace Equation.
2. Differentiate between linear and non-linear PDE.
3. Write general formula for numerical integration.
4. Define cyclic group.
5. Define symmetric operation. **(4×1=4)**

PART – B

Answer **any seven** questions. **Each** question carries **2** marks.

6. Identify the type of following Quasi-linear PDE
   a) $2xyU_{xy} + xU_y + yU_x = 0$
   b) $U_{xx} + U_{xy} + 5U_{yy} + 6U_x = 0$.
7. Find the characteristics of $3U_{xx} + 10U_{xy} + 3U_{yy} = 0$.
8. Give one dimensional heat equation with boundary conditions. Give solution of the problem by Fourier Series.
9. Find the deflection of vibrating string of unit length having fixed ends with initial velocity zero and deflection $f(x) = k(\sin x - \sin 2x)$.

P.T.O.

10. Find the function U(x, t) satisfies the initial value problem.

$$\frac{\partial^2 U}{\partial t^2} = \frac{\partial^2 U}{\partial x^2}, x \in \mathbb{R},\ t > 0,\ U(x,0) = x,\ U_t(x,0) = 0.$$

11. Evaluate $\int_0^6 xe^{-0.5x}dx$ using the trapezoidal rule with n = 3 to 3 decimal places.

12. Solve the initial-value problem $y' = y - x,\ y(0) = \frac{1}{2}$ using modified Euler's method with h = 0.1 to obtain an approximation to y(1).

13. Solve the initial-value problem y′ = –y, y(0) = 1 using Euler's method with h = 0.01 to obtain an approximation to y(0.04).

14. Let G be a group with $(ab)^2 = a^2b^2$ for every a, b in G. Show that G is abelian.

15. Show that the three reflection of NH_3 constitute a class. **(7×2=14)**

PART – C

Answer **any four** questions. **Each** question carries **3** marks.

16. Solve using the method of separation of variables

$$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0.$$

17. Give the condition when Quasi-linear equation

$AU_{xx} + BU_{xy} + CU_{yy} = F(x, y, u, u_x, u_y)$ is

i) Hyperbolic ii) Elliptic iii) Parabolic.

18. Approximate $\int_0^1 \sqrt{1+x^3}\,dx$ using the Trapezoidal rule with n = 5 to 3 decimal places.

19. Solve the differential equation y′ = x + y with conditions y(0) = 1 by Taylor series method. Hence find the value of y at x = 0.1 and x = 0.2.

20. List the five type of symmetry elements of molecule.

21. Prove that in any abelian group each element is in a class by itself.

22. Give multiplication table of a group of order 3. **(4×3=12)**

PART – D

Answer **any two** questions. **Each** question carries **5** marks.

23. Solve the one dimensional wave equation, $U_{tt} = c^2U_{xx}$, satisfying $U(0, t) = U(l, t) = 0\ \forall t$ and initial deflection is given by

$$f(x) = \begin{cases} \sin\left(\frac{\pi x}{c}\right) & 0 \le x \le c \\ 0 & \text{otherwise} \end{cases} \text{ and } U_t(x, 0) = 0.$$

24. Find the first four terms of the Taylor expansion of the solution of $y' = x + y^2$, $y(0) = 1$ about $x_0 = 0$.

25. Given that $\frac{dy}{dx} = y - x$ where $y(0) = 2$, find $y(0.1)$ and $y(0.4)$ correct to four decimal places, using Runge-Kutta second order formula. Take $h = 0.1$.

26. From the group multiplication table for water molecule by verifying the properties of a group. **(2×5=10)**

K23U 0881

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CBCSS – Supplementary) Examination, April 2023
(2017 and 2018 Admissions)
COMPLEMENTARY COURSE IN MATHEMATICS
4C04MAT-CH : Mathematics for Chemistry – IV

Time : 3 Hours Max. Marks : 40

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Define divergence of a vector.
2. State Green's theorem in the plane.
3. Define the term irrotational.
4. Write Legrange's Interpolation formula.

SECTION – B

Answer **any 7** questions from among the questions **5** to **13**. These questions carry **2** marks **each**.

5. Find the curl of $\vec{v} = yz\hat{i} + 3zx\hat{j} + z\hat{k}$.

6. Using Stoke's theorem evaluate $\iint_S (\text{curl}\,\vec{F})\cdot\hat{n}\,dA$ where $\vec{F} = z^2\hat{i} + 5x\hat{j}$ and S is the square $0 \le x \le 1$, $0 \le y \le 1$, $z = 1$.

7. Evaluate $\iint_S \vec{F}\cdot\hat{n}\,dA$ where $\vec{F} = x\hat{j}$ and S is the portion of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

P.T.O.

8. Using Divergence theorem, evaluate $\iint_S \vec{F}\cdot\hat{n}\,dA$ where $\vec{F} = x^2\,\hat{i} + z^2\,\hat{k}$ and S is the surface of the cube $|x| \le 1$, $|y| \le 3$, $|z| \le 2$.

9. Use Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$.

10. Find the missing term in the following table :

x	0	1	2	3	4
y	1	3	9	–	81

11. The function y = sinx is tabulated below. Using Lagrange's interpolation formula, find the value of $\sin\frac{\pi}{6}$.

x	0	$\pi/4$	$\pi/2$
y = sinx	0	0.70711	1.0

12. Given $\frac{dy}{dx} - 1 = xy$ and y(0) = 1, obtain Taylor series for y(x).

13. Using Euler's method, solve the initial-value problem
$\frac{dy}{dx} = x + y,\ y(0) = 0$.
Choose h = 0.2 and compute y(0.6).

SECTION – C

Answer **any 4** questions from among the questions **14** to **19**. These questions carry **3** marks **each**.

14. Show that the curvature of a circle of radius a is $\frac{1}{a}$.

15. Find the curvature of $y = x^2$.

16. Using Green's theorem, evaluate $\oint_C \vec{F}\cdot dr$ where $\vec{F} = 3y^2\,\hat{i} + x - y^4\,\hat{j}$, C is the square with vertices (1, 1), (– 1, 1), (– 1, – 1), (1, – 1).

17. The table below gives the values of tanx for $0.10 \le x \le 0.30$

x	0.10	0.15	0.20	0.25	0.30
y = tanx	0.1003	0.1511	0.2027	0.2553	0.3093

Find tan0.12.

18. Find the cubic polynomial which takes the following values :

y(1) = 24, y(3) = 120, y(5) = 336 and y(7) = 720.

19. Given the differential equation.

$$\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$$

with the initial condition y = 0 when x = 0, use Picard's method to obtain y for x = 0.5 correct to 3 decimal places.

SECTION – D

Answer **any 2** questions from among the questions **20** to **23**. These questions carry **5** marks **each.**

20. For any twice continuously differentiable vector functions $\vec{u}$ and $\vec{v}$, show that $\text{curl}\,(\vec{u} + \vec{v}) = \text{curl}\,\vec{u} + \text{curl}\,\vec{v}$.

21. Verify Stoke's theorem for F = [y, z, x] over the paraboloid S : $z = 1 - (x^2 + y^2)$, $z \ge 0$.

22. Calculate the first and second derivatives of the function tabulated below at the point x = 2.2 and also $\frac{dy}{dx}$ at x = 2.0.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

23. Given the differential equation $y'' - xy' - y = 0$ with the conditions y(0) = 1 and $y'(0) = 0$, use Taylor's series method to determine the value of y(0.1).

K24U 0731

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/ Improvement) Examination, April 2024
(2019 to 2022 Admissions)
COMPLEMENTARY ELECTIVE COURSE IN MATHEMATICS
4C04 MAT – PH : Mathematics for Physics – IV

Time : 3 Hours Max. Marks : 40

PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark.

1. Define a linear partial differential equation.
2. Define a gradient field of a differential function f(x, y, z).
3. Give an example of an non-orientable surface.
4. State trapezoidal rule.
5. State the fundamental theorem of line integrals. **(4×1=4)**

PART – B

Answer **any seven** questions. **Each** question carries **2** marks.

6. Verify that u = e^xcosy, e^xsiny is a solution of the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$.
7. Verify that u = v(x) + w(y) with any v and w satisfies the partial differential equation $u_{xy} = 0$.
8. Integrate $f(x, y) = \frac{x^3}{y}$ over the curve C : $y = x^2/2, 0 \le x \le 2$.
9. Prove that curl grad f = $\overline{0}$.
10. State Green's theorem.

P.T.O.

11. Find a parametrization of the sphere $x^2 + y^2 + z^2 = a^2$.

12. Solve $y' = -y$ with the condition that $y(0) = 1$ by Euler's method.

13. Explain the method of solution of differential equation $y'(x, y) = f(x, y)$ with the initial condition $y(x_0) = y_0$ by Taylor series.

14. Explain Euler's method to find the solution of the differential equation.

15. Find the divergence of the vector field $\overline{F}(x, y, z) = z\hat{j}$.

16. Find the work done by the force field $\overline{F} = x\hat{i} + y\hat{j} + z\hat{k}$ in moving an object along the curve C parametrized by $\overline{r}(t) = \cos(\pi t)\hat{i} + t^2\hat{j} + \sin(\pi t)\hat{k},\ 0 \le t \le 1$. **(7×2=14)**

PART – C

Answer **any four** questions. **Each** question carries **3** marks.

17. Solve $u_{xx} + 2u_{xy} + u_{yy} = 0$.

18. If u_1 and u_2 are solutions of $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ in some region R. Prove that $u = c_1u_1 + c_2u_2$ is also a solution of the above partial differential equation.

19. Find the flux of $\overline{\overline{F}} = (x - y)\hat{i} + x\hat{j}$ across the circle $x^2 + y^2 = 1$ in the xy-plane.

20. Integrate $G(x, y, z) = \sqrt{1 - x^2 - y^2}$ over the "football" surface S formed by rotating the curves $x = \cos z$,$y = 0$, $-\frac{\pi}{2} \le z \le \frac{\pi}{2}$ around the z-axis.

21. A solid of revolution is formed by rotating about the x-axis the area between the x-axis, the lines x = 0 and x = 1, and a curve through the points with the following coordinates :

X	**Y**
0.00	1.0000
0.25	0.9896
0.50	0.9589
0.75	0.9089
1.00	0.8415

22. Evaluate $y = \int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx$ using Simpson's $\frac{1}{3}$ rule with $h = \frac{\pi}{12}$.

23. Find the surface area of a sphere of radius a. **(4×3=12)**

PART – D

Answer **any two** questions. **Each** question carries **5** marks.

24. Find the solution of the one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, subject to the initial conditions $u(x, 0) = f(x)$, $u_t(x, 0) = g(x)$, $(0 \le x \le L)$.

25. Verify both forms of Green's theorem for the vector field $\bar{F}(x, y) = (x - y)\hat{i} + x\hat{j}$ and the region R bounded by the unit circle C : $\bar{r}(t) = \cos t\, \hat{i} + \sin t\, \hat{j}$, $0 \le t \le 2\pi$.

26. Given $\frac{dy}{dx} = y - x$, where $y(0) = 2$, find $y(0.1)$ and $y(0.2)$ correct to four decimal places.

27. Find the outward flux of the field $\bar{F} = (y - x)\hat{i} + (z - y)\hat{j} + (y - x)\hat{k}$ across the boundary of the cube D bounded by the planes $x = \pm 1$, $y = \pm 1$ and $z = \pm 1$. **(2×5=10)**