K23U 1126

Reg. No. : ....

Name : ....

IV Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
4B04 MAT : Number Theory and Applications of Integrals

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** out of five questions. **Each** question carries **1** mark.

1. Define a prime number.

2. State Euclidian lemma.

3. When we can say that the existence of solution Diophantine equation of the form $ax + by = c$?

4. State Wilson's theorem.

5. Show that for any integers a, n, $a \equiv a(\text{mod} n)$. **(4×1=4)**

PART – B
(Short Essay Type)

Answer **any eight** out of eleven questions. **Each** question carries **2** marks.

6. If $a|b$, then show that $a|bc$, for any integer c.

7. Find gcd(24, 138) using Euclidian algorithm.

8. Show that, if p is a prime and $p|ab$, then $p|a$ or $p|b$.

9. If $a \equiv b(\text{mod } n)$, prove that $\gcd(a, n) = \gcd(b, n)$.

P.T.O.

10. State Fermat's little theorem.

11. Evaluate $\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$.

12. Find the area of the region bounded above by y = x + 6 bounded below by $y = x^2$, and bounded on the sides by the lines x = 0 and x = 2.

13. Define volume problem.

14. Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$, over the interval [1, 4] is revolved about x-axis.

15. Find the arc length of the spiral $r = e^{\theta}$ between $\theta = 0$ and $\theta = 2 + \pi$.

16. Find the area of the surface generated by y = 7x, $0 \le x \le 1$, revolving about x-axis. **(8×2=16)**

PART – C
(Essay Type)

Answer **any four** out of seven questions. **Each** question carries **4** marks.

17. Solve the Diophantine equation 172x + 20y = 1000.

18. Express 6 as a linear combination of 12378 and 3054.

19. Find 2^{340}(mod 341).

20. Find the area of the region enclosed by $x = y^2$ and y = x – 2.

21. Find the area of the region enclosed by the rose curve $r = \cos 2\theta$.

22. Find the arc length of the curve $y = x^{\frac{3}{2}}$, from (1,1) to $(2, 2\sqrt{2})$.

23. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between x = 0 and x = 1 about the x-axis. **(4×4=16)**

PART – D
(Long Essay Type)

Answer **any two** out of four questions. **Each** question carries **6** marks.

24. Let x_0, y_0 is any particular solution of the Diophantine equation of the form $ax + by = c$, then show that all other solutions can be represented by $x = x_0 + \left(\frac{b}{d}\right) t$, $y = y_0 + \left(\frac{a}{d}\right) t$, where $d = \gcd(a, b)$.

25. Use Euler's theorem, evaluate $2^{100000} \pmod{77}$.

26. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, using integration.

27. Find the volume of the solid generated when the region under $y = x^2$ over the interval $[0, 2]$ is revolved about the line $y = -1$. **(2×6=12)**

K24U 0730

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CORE COURSE IN MATHEMATICS
4B04 MAT : Number Theory and Applications of Integrals

Time : 3 Hours | Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark.

1. State Division algorithm.
2. Write the positive divisors of 30.
3. What do you mean by least common multiple of two integers ?
4. State Fundamental theorem of Arithmetic.
5. State Wilson's theorem. **(4×1=4)**

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks.

6. Let a, b and c be three integers. If a|b and b|c, then prove that a|c.
7. If k > 0, then prove that gcd(ka, kb) = k gcd(a, b).
8. Using division algorithm, find the gcd of 143 and 227.
9. If a ≡ b (mod n) and b ≡ c (mod n), then prove that a ≡ c (mod n).

P.T.O.

10. Prove that 41 divides $2^{20} - 1$.

11. Evaluate $\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$.

12. Let f be continuous on the symmetric interval [– a, a]. If f is even, prove that $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

13. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

14. What are the steps to find the volume of a solid using area of cross section ?

15. The base of a solid is the region bounded by the graphs of y = 3x, y = 6 and x = 0. The cross-sections perpendicular to the x- axis are rectangles of height 10. Find the volume of the solid.

16. The circle $x^2 + y^2 = a^2$ is rotated about the x-axis to generate a sphere. Find its volume. **(8×2=16)**

PART – C

Answer **any 4** questions from this Part. **Each** question carries **4** marks.

17. Show that the expression $\frac{a(a^2+2)}{3}$ is an integer for $a \geq 1$.

18. Find the remainder when the sum 1! + 2! + ... 100! is divisible by 12.

19. Using Euler's theorem, prove that for any integer a, $a^{37} \equiv a \pmod{1729}$.

20. Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below the x-axis and the line y = x – 2 by integrating with respect to x.

21. Find the length of the curve $y = \frac{x^3}{12} + \frac{1}{x}, 1 \leq x \leq 4$.

22. The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y-axis to generate a solid. Find the volume of the solid.

23. Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$. **(4×4=16)**

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks.

24. Let a and b two integers, not both of which are zero. Prove that there exist integers x and y such that $\gcd(a, b) = ax + by$.

25. Prove that the quadratic congruence $x^2 + 1 \equiv 0 \pmod{p}$, where p is an odd prime, has a solution if and only if $p \equiv 1 \pmod{p}$.

26. Find the area of the segment cut off from the parabola $x^2 = 8y$ by the line $x - 2y + 8 = 0$.

27. Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$, about the x-axis. **(2×6=12)**