ELECTROMAGNETIC THEORY

NET/JRF–(JUNE-2011)

Q1. The electrostatic potential $V(x, y)$ in free space in a region where the charge density ρ is zero is given by $V(x,y)=4e^{2x}+f(x)-3y^2$. Given that the x-component of the electric field E_x, and V are zero at the origin, $f(x)$ is

(a) $3x^2-4e^{2x}+8x$ (b) $3x^2-4e^{2x}+16x$

(c) $4e^{2x}-8$ (d) $3x^2-4e^{2x}$

Ans. : (d)

Solution: $V=4e^{2x}+f(x)-3y^2$. Since $\rho=0\Rightarrow\nabla^2V=0\Rightarrow 16e^{2x}+f''(x)-6=0$.

Since $E_x=0$ at origin $\Rightarrow \vec{E}=-\vec{\nabla}V\Rightarrow E_x=-\left[8e^{2x}+f'(x)\right]$

$E_x(0,0)=-\left[8+f'(0)\right]=0\Rightarrow f'(0)=-8$.

Since $V(0,0)=0\Rightarrow 4+f(0)=0\Rightarrow f(0)=-4$

Solve equation $16e^{2x}+f''(x)-6=0\Rightarrow f''(x)=6-16e^{2x}\Rightarrow f'(x)=6x-8e^{2x}+c_1$, since $f'(0)=-8+c_1=-8\Rightarrow c_1=0$.

Again Integrate $f'(x)=6x-8e^{2x}\Rightarrow f(x)=3x^2-4e^{2x}+c_2$

since $f(0)=-4+c_2=-4\Rightarrow c_2=0$. Thus $f(x)=3x^2-4e^{2x}$

Q2. For constant uniform electric and magnetic field $\vec{E}=\vec{E}_0$ and $\vec{B}=\vec{B}_0$, it is possible to choose a gauge such that the scalar potential ϕ and vector potential $\vec{A}$ are given by

(a) $\phi=0$ *and* $\vec{A}=\frac{1}{2}\left(\vec{B}_0\times\vec{r}\right)$ (b) $\phi=-\vec{E}_0\cdot\vec{r}$ *and* $\vec{A}=\frac{1}{2}\left(\vec{B}_0\times\vec{r}\right)$

(c) $\phi=-\vec{E}_0\cdot\vec{r}$ *and* $\vec{A}=0$ (d) $\phi=0$ *and* $\vec{A}=-\vec{E}_0t$

Ans. : (a)

Solution: Let $\vec{E}=E_0(\hat{x}+\hat{y}+\hat{z})$ and $\vec{B}=B_0(\hat{x}+\hat{y}+\hat{z})$ since they are constant vector.

Lorentz Gauge condition is $\vec{\nabla}\cdot\vec{A}=-\mu_0\varepsilon_0\frac{\partial\phi}{\partial t}$

$\left\{\text{since } \vec{B}\times\vec{r}=B_0(z-y)\hat{x}-B_0(z-x)\hat{y}+B_0(y-x)\hat{z}\right\}$

(a) $\frac{\partial\phi}{\partial t}=0$ and $\vec{\nabla}\cdot\vec{A}=0$ (b) $\frac{\partial\phi}{\partial t}\neq 0,$ and $\vec{\nabla}\cdot\vec{A}=0$

(c) $\frac{\partial\phi}{\partial t}\neq 0$ and $\vec{\nabla}\cdot\vec{A}=0$ (d) $\frac{\partial\phi}{\partial t}=0$ and $\vec{\nabla}\cdot\vec{A}\neq 0$

Q3. A plane electromagnetic wave is propagating in a lossless dielectric. The electric field is given by

$$\vec{E}(x,y,z,t)=E_0(\hat{x}+A\hat{z})\exp\left[ik_0\left\{-ct+\left(x+\sqrt{3}\,z\right)\right\}\right],$$

where c is the speed of light in vacuum, E_0, A and k_0 are constant and $\hat{x}$ and $\hat{z}$ are unit vectors along the x- and z-axes. The relative dielectric constant of the medium ε_r and the constant A are

(a) $\varepsilon_r=4$ and $A=-\frac{1}{\sqrt{3}}$ (b) $\varepsilon_r=4$ and $A=+\frac{1}{\sqrt{3}}$

(c) $\varepsilon_r=4$ and $A=\sqrt{3}$ (d) $\varepsilon_r=4$ and $A=-\sqrt{3}$

Ans. : (a)

Solution: $\vec{E}(x,y,z,t)=E_0(\hat{x}+A\hat{z})\exp\left[ik_0\left\{-ct+\left(x+\sqrt{3}z\right)\right\}\right]$.

Comparing with term $e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)}\Rightarrow\vec{k}=k_0\left(\hat{x}+\sqrt{3}\hat{z}\right)$ and $\omega=k_0c$.

Since $v=\frac{\omega}{k}=\frac{k_0c}{\sqrt{k_0^2+3k_0^2}}=\frac{c}{2}\Rightarrow$ Refractive index $n=\sqrt{\varepsilon_r}=2\Rightarrow\varepsilon_r=4$.

Since $\vec{k}\cdot\hat{n}=0\Rightarrow k_0\left(\hat{x}+\sqrt{3}\hat{z}\right)\cdot\left(\hat{x}+A\hat{z}\right)=0\Rightarrow k_0\left(1+A\sqrt{3}\right)=0\Rightarrow A=-\frac{1}{\sqrt{3}}$

Q4. A static, spherically symmetric charge distribution is given by $\rho(r)=\frac{A}{r}e^{-Kr}$ where A and K are positive constants. The electrostatic potential corresponding to this charge distribution varies with r as

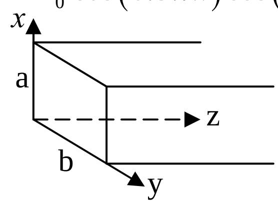
(a) re^{-Kr} (b) $\frac{1}{r}e^{-Kr}$ (c) $\frac{1}{r^2}e^{-Kr}$ (d) $\frac{1}{r}\left(1-e^{-Kr}\right)$

Ans. : (b)

Solution: since $\nabla^2V=-\rho/\varepsilon_0$

$\nabla^2 V$ must be proportional to $\frac{A}{r}e^{-kr}$, where $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right)$.

Q5. The magnetic field of the TE_{11} mode of a rectangular waveguide of dimensions $a \times b$ as shown in the figure is given by $H_z = H_0 \cos(0.3\pi x)\cos(0.4\pi y)$, where x and y are in cm.



A. The dimensions of the waveguide are

(a) $a = 3.33 cm,\ b = 2.50\ cm$

(b) $a = 0.40\ cm, b = 0.30\ cm$

(c) $a = 0.80\ cm, b = 0.60\ cm$

(d) $a = 1.66 cm,\ b = 1.25\ cm$

Ans. : (a)

Solution: Since $H_z = H_0 \cos(0.3\pi x)\cos(0.4\pi y)$

$$\Rightarrow \frac{m\pi}{a} = 0.3\pi \quad \text{where } m = 1 \text{ and } \frac{n\pi}{b} = 0.4\pi \quad \text{where } n = 1$$

$$\Rightarrow a = 3.33 cm,\ b = 2.50 cm$$

B. The entire range of frequencies f for which the TE_{11} mode will propagate is

(a) $6.0 GHz < f < 12.0 GHz$

(b) $7.5 GHz < f < 9.0 GHz$

(c) $7.5 GHz < f < 12.0 GHz$

(d) $7.5 GHz < f$

Ans. : (d)

Solution: $f_{m,n} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \Rightarrow f_{1,1} = \frac{c}{2}\sqrt{\frac{1}{a^2} + \frac{1}{b^2}} = 7.5 GH_z$.

For propagation, frequency of incident wave must be greater than cutoff frequency.

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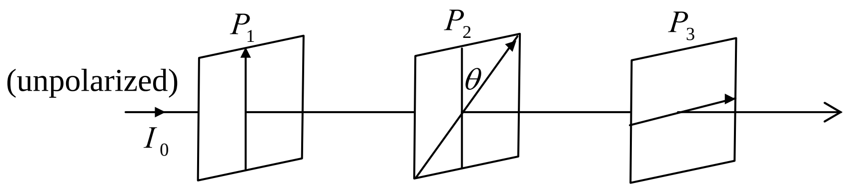
Q6. Consider three polarizer's P_1, P_2 and P_3 placed along an axis as shown in the figure.



The pass axis of P_1 and P_3 are at right angles to each other while the pass axis of P_2 makes an angle θ with that of P_1. A beam of unpolarized light of intensity I_0 is incident on P_1 as shown. The intensity of light emerging from P_3 is

(a) 0 (b) $\frac{I_0}{2}$ (c) $\frac{I_0}{8}\sin^2 2\theta$ (d) $\frac{I_0}{4}\sin^2 2\theta$

Ans. : (c)

Solution: $I = I_0\cos^2\theta$ (Malus Law)

$$\Rightarrow I_1 = \frac{I_0}{2}, \quad I_2 = \frac{I_0}{2}\cos^2\theta, \quad I_3 = \frac{I_0}{2}\cos^2\theta \times \cos^2(90-\theta) = \frac{I_0}{8}\sin^2 2\theta.$$

Q7. Four equal point charges are kept fixed at the four vertices of a square. How many neutral points (i.e. points where the electric field vanishes) will be found inside the square?

(a) 1 (b) 4 (c) 5 (d) 7

Ans. : (a)

Solution: Inside the square, there is only one point where field vanishes.

Q8. A static charge distribution gives rise to an electric field of the form $\vec{E} = \alpha\left(1 - e^{-r/R}\right)\frac{\hat{r}}{r^2}$, where α and R are positive constants. The charge contained within a sphere of radius R, centred at the origin is

(a) $\pi\alpha\varepsilon_0\frac{e}{R^2}$ (b) $\pi\alpha\varepsilon_0\frac{e^2}{R^2}$ (c) $4\pi\alpha\varepsilon_0\frac{R}{e}$ (d) $\pi\alpha\varepsilon_0\frac{R^2}{e}$

Ans. : None of the options given are correct

Solution: $Q_{enc} = \varepsilon_0\oint\vec{E}\cdot d\vec{a} = \alpha\varepsilon_0\int\left(1-e^{-r/R}\right)\frac{\hat{r}}{r^2}\cdot\left(r^2\sin\theta d\theta d\phi\hat{r}\right) = \alpha\varepsilon_0 \times \int_0^{\pi}\int_0^{2\pi}\left(1-e^{-r/R}\right)\sin\theta d\theta d\phi$

at $r = R$, $Q_{enc} = 4\pi\alpha\varepsilon_0\left(1-\frac{1}{e}\right)$. So none of the options given are correct.

Q9. In a Young's double slit interference experiment, the slits are at a distance $2L$ from each other and the screen is at a distance D from the slits. If a glass slab of refractive index μ and thickness d is placed in the path of one of the beams, the minimum value of d for the central fringe to be dark is

(a) $\frac{\lambda D}{(\mu-1)\sqrt{D^2+L^2}}$ (b) $\frac{\lambda D}{(\mu-1)L}$

(c) $\frac{\lambda}{(\mu-1)}$ (d) $\frac{\lambda}{2(\mu-1)}$

Ans. : (d)

Solution: For central fringe to be dark, $(\mu-1)d = \frac{n\lambda}{2} \Rightarrow d = \frac{\lambda}{2(\mu-1)}$

Q10. Consider a solenoid of radius R with n turns per unit length, in which a time dependent current $I = I_0 \sin\omega t$ (where $\omega R/c << 1$) flows. The magnitude of the electric field at a perpendicular distance $r < R$ from the axis of symmetry of the solenoid, is

(a) 0 (b) $\frac{1}{2r}\omega\mu_0 n I_0 R^2 \cos\omega t$

(c) $\frac{1}{2}\omega\mu_0 n I_0 r \sin\omega t$ (d) $\frac{1}{2}\omega\mu_0 n I_0 r \cos\omega t$

Ans. : (d)

Solution: $\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \overline{B}}{\partial t}\cdot d\vec{a}$; $\left(\vec{B} = \mu_0 n I(t)\hat{z}\right)$.

$$\Rightarrow |\vec{E}|\times 2\pi r = -\mu_0 n \frac{dI}{dt}\int_{r'=0}^{r} 2\pi r' dr' = -\mu_0 n \times I_0 \omega \cos\omega t \times \frac{2\pi r^2}{2}$$

$$\Rightarrow |\vec{E}| = -\frac{1}{2}\times \omega\mu_0 n I_0 r \cos\omega t$$

Q11. A constant electric current I in an infinitely long straight wire is suddenly switched on at $t = 0$. The vector potential at a perpendicular distance r from the wire is given by $\vec{A} = \frac{\hat{k}\mu_0 I}{2\pi}\ln\left[\frac{1}{r}\left(ct + \sqrt{c^2t^2 - r^2}\right)\right]$. The electric field at a distance $r\,(< ct)$ is

(a) 0 (b) $\frac{\mu_0 I}{2\pi t}\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}\right)$

(c) $\frac{c\mu_0 I}{2\pi\sqrt{c^2t^2-r^2}}\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$ (d) $-\frac{c\mu_0 I}{2\pi\sqrt{c^2t^2-r^2}}\hat{k}$

Ans. : (d)

Solution: $\vec{E}=-\vec{\nabla}\phi-\frac{\partial\vec{A}}{\partial t}=-\frac{\partial\vec{A}}{\partial t}\Rightarrow\vec{E}=-\frac{\mu_0 I}{2\pi}\frac{r}{\left(ct+\sqrt{c^2t^2-r^2}\right)}\left[\frac{1}{r}\left(c+\frac{2c^2t}{2\sqrt{c^2t^2-r^2}}\right)\right].$

$$\Rightarrow\vec{E}=\frac{-c\mu_0 I}{2\pi\sqrt{c^2t^2-r^2}}\hat{k}$$

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Q12. The magnetic field corresponding to the vector potential $\vec{A}=\frac{1}{2}\vec{F}\times\vec{r}+\frac{10}{r^3}\vec{r}$, where $\vec{F}$ is a constant vector, is

(a) $\vec{F}$ (b) $-\vec{F}$ (c) $\vec{F}+\frac{30}{r^4}\vec{r}$ (d) $\vec{F}-\frac{30}{r^4}\vec{r}$

Ans. : (a)

Solution: $\vec{B}=\vec{\nabla}\times\vec{A}=\frac{1}{2}\left[\vec{\nabla}\times\left(\vec{F}\times\vec{r}\right)\right]+10\left(\vec{\nabla}\times\frac{\vec{r}}{r^3}\right)$. Since $\vec{F}$ is a constant vector, let

$$\vec{F}=F_0\left(\hat{x}+\hat{y}+\hat{z}\right),\ \vec{F}\times\vec{r}=\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\F_0&F_0&F_0\\x&y&z\end{vmatrix}=\hat{x}(z-y)F_0-\hat{y}(z-x)F_0+\hat{z}(y-x)F_0$$

$$\vec{\nabla}\times\left(\vec{F}\times\vec{r}\right)=\begin{vmatrix}\hat{x}&\hat{y}&\hat{z}\\\frac{\partial}{\partial x}&\frac{\partial}{\partial y}&\frac{\partial}{\partial z}\\(z-y)F_0&(x-z)F_0&(y-x)F_0\end{vmatrix}=\hat{x}[F_0+F_0]-\hat{y}[-F_0-F_0]+\hat{z}[F_0+F_0]=2F_0\left(\hat{x}+\hat{y}+\hat{z}\right)$$

$\Rightarrow\frac{1}{2}\left[\vec{\nabla}\times\left(\vec{F}\times\vec{r}\right)\right]=F_0\left(\hat{x}+\hat{y}+\hat{z}\right)=\vec{F}$, $\vec{\nabla}\times\frac{\vec{r}}{r^3}=0$. Thus $\vec{B}=\vec{F}$

Q13. An electromagnetic wave is incident on a water-air interface. The phase of the perpendicular component of the electric field, $E_\perp$, of the reflected wave into the water is found to remain the same for all angles of incidence. The phase of the magnetic field H

(a) does not change (b) changes by $3\pi/2$

(c) changes by $\pi/2$ (d) changes by π

Ans. : (d)

Q14. The magnetic field at a distance R from a long straight wire carrying a steady current I is proportional to

(a) IR (b) I/R^2 (c) I^2/R^2 (d) I/R

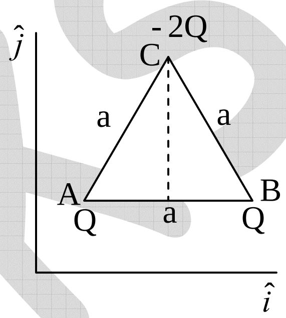
Ans. : (d)

Q15. Which of the following questions is Lorentz invariant?

(a) $|\vec{E}\times\vec{B}|^2$ (b) $|\vec{E}|^2-|\vec{B}|^2$ (c) $|\vec{E}|^2+|\vec{B}|^2$ (d) $|\vec{E}|^2|\vec{B}|^2$

Ans. : (b)

Q16. Charges Q, Q and $-2Q$ are placed on the vertices of an equilateral triangle ABC of sides of length a, as shown in the figure. The dipole moment of this configuration of charges, irrespective of the choice of origin, is



(a) $+2aQ\,\hat{i}$

(b) $+\sqrt{3}aQ\,\hat{j}$

(c) $-\sqrt{3}aQ\,\hat{j}$

(d) 0

Ans. : (c)

Solution: Let coordinates of A is (l, m), then

$$\vec{p}=q_i\vec{r}_i'=Q\left[l\hat{i}+m\hat{j}\right]+Q\left[(l+a)\hat{i}+m\hat{j}\right]-2Q\left[\left(l+\frac{a}{2}\right)\hat{i}+\left(m+\frac{\sqrt{3}a}{2}\right)\hat{j}\right]$$

$$\vec{p}=Q\left[l\hat{i}+m\hat{j}\right]+Q\left[(l+a)\hat{i}+m\hat{j}\right]-Q\left[(2l+a)\hat{i}+\left(2m+\sqrt{3}a\right)\hat{j}\right]\Rightarrow\vec{p}=-\sqrt{3}aQ\hat{j}$$

Q17. The vector potential $\vec{A}$ due to a magnetic moment $\vec{m}$ at a point $\vec{r}$ is given by $\vec{A}=\frac{\vec{m}\times\vec{r}}{r^3}$.

If $\vec{m}$ is directed along the positive z-axis, the x- component of the magnetic field, at the point $\vec{r}$, is

(a) $\frac{3myz}{r^5}$ (b) $-\frac{3mxy}{r^5}$ (c) $\frac{3mxz}{r^5}$ (d) $\frac{3m\left(z^2-xy\right)}{r^5}$

Ans. : (c)

Solution: $\vec{m}=m\hat{z}$ and $\vec{B}=\vec{\nabla}\times\vec{A}=\frac{m}{r^3}\left(2\cos\theta\hat{r}+\sin\theta\hat{\theta}\right)=\frac{1}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r}-\vec{m}\right]$

$$\vec{B}=\frac{1}{r^3}\left[3m\hat{z}\cdot\left(\frac{x\hat{x}+y\hat{y}+z\hat{z}}{r}\right)\frac{\vec{r}}{r}-m\hat{z}\right]\Rightarrow\qquad B_x=\frac{3mxz}{r^5}$$

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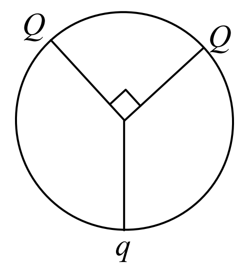
Q18. Three charges are located on the circumference of a circle of radius R as shown in the figure below. The two charges Q subtend an angle 90^0 at the centre of the circle. The charge q is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q?



(a) $q/\sqrt{2}$ (b) $\sqrt{2}q$ (c) $2q$ (d) $4q$

Ans. : (a)

Solution: $E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$ and $E_3 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

Resultant of E_1 *and* E_2 is $E = \sqrt{E_1^2 + E_2^2} = \sqrt{2}E_1$, Thus $E_3 = E \Rightarrow Q = \frac{q}{\sqrt{2}}$

Q19. Consider a hollow charged shell of inner radius a and outer radius b. The volume charge density is $\rho(r) = \frac{k}{r^2}$ (k is constant) in the region $a < r < b$. The magnitude of the electric field produced at distance $r > a$ is

(a) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for all $r > a$,

(b) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{kb}{\varepsilon_0 r^2}$ for $r > b$

(c) $\frac{k(r-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > b$

(d) $\frac{k(r-a)}{\varepsilon_0 a^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > b$

Ans. : (c)

Solution: For $r > a$: $\oint \vec{E}.d\vec{a} = E(4\pi r^2) = \frac{1}{\varepsilon_0}Q_{enc} = \frac{1}{\varepsilon_0}\int \rho dV = \frac{1}{\varepsilon_0}\int \frac{k}{|\vec{r}|^2}|\vec{r}|^2 \sin\theta dr d\theta d\phi$

$$E(4\pi r^2) = \frac{4\pi k}{\varepsilon_0}\int_a^r d\vec{r} = \frac{4\pi k}{\varepsilon_0}(r-a) \Rightarrow \vec{E} = \frac{k}{\varepsilon_0}\left(\frac{r-a}{r^2}\right)\hat{r}$$

For $r > b$: $E4\pi r^2 = \frac{4\pi k}{\varepsilon_0}\int_a^b dr = \frac{4\pi k}{\varepsilon_0}(b-a) \Rightarrow \vec{E} = \frac{k}{\varepsilon_0}\left(\frac{b-a}{r^2}\right)\hat{r}$

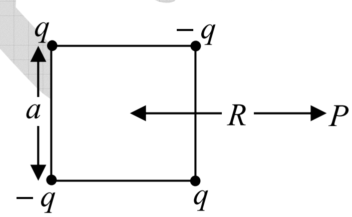
Q20. Consider the interference of two coherent electromagnetic waves whose electric field vectors are given by $\vec{E}_1 = \hat{i}E_0 \cos\omega t$ and $\vec{E}_2 = \hat{j}E_0 \cos(\omega t + \varphi)$ where φ is the phase difference. The intensity of the resulting wave is given by $\frac{\varepsilon_0}{2}\langle E^2\rangle$, where $\langle E^2\rangle$ is the time average of E^2. The total intensity is

(a) 0 (b) $\varepsilon_0 E_0^2$ (c) $\varepsilon_0 E_0^2 \sin^2\varphi$ (d) $\varepsilon_0 E_0^2 \cos^2\varphi$

Ans. : (a)

Solution: Since waves are polarized in perpendicular direction hence there will be no interference.

Q21. Four charges (two $+q$ and two $-q$) are kept fixed at the four vertices of a square of side a as shown. At the point P which is at a distance R from the centre $(R >> a)$, the potential is proportional to



(a) $1/R$ (b) $1/R^2$

(c) $1/R^3$ (d) $1/R^4$

Ans. : (c)

Solution: Given configuration is quadrupole.

Q22. A point charges q of mass m is kept at a distance d below a grounded infinite conducting sheet which lies in the xy - plane. For what value of d will the charge remains stationary?

(a) $q/4\sqrt{mg\pi\varepsilon_0}$ (b) $q/\sqrt{mg\pi\varepsilon_0}$

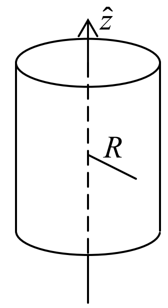
(c) There is no finite value of d (d) $\sqrt{mg\pi\varepsilon_0}/q$

Ans. : (a)

Solution: There is attractive force between point charge q and grounded conducting sheet that can be calculate from method of images i.e. $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2d)^2} = mg \Rightarrow d = \frac{q}{4\sqrt{mg\pi\varepsilon_0}}$

Q23. An infinite solenoid with its axis of symmetry along the z-direction carries a steady current I.

The vector potential $\vec{A}$ at a distance R from the axis



(a) is constant inside and varies as R outside the solenoid

(b) varies as R inside and is constant outside the solenoid

(c) varies as $\frac{1}{R}$ inside and as R outside the solenoid

(d) varies as R inside and as $\frac{1}{R}$ outside the solenoid

Ans. : (d)

Q24. Consider an infinite conducting sheet in the xy-plane with a time dependent current density $Kt\hat{i}$, where K is a constant. The vector potential at (x, y, z) is given by $\vec{A} = \frac{\mu_0 K}{4c}(ct - z)^2 \hat{i}$. The magnetic field $\vec{B}$ is

(a) $\frac{\mu_0 Kt}{2}\hat{j}$ (b) $-\frac{\mu_0 Kz}{2c}\hat{j}$ (c) $-\frac{\mu_0 K}{2c}(ct - z)\hat{i}$ (d) $-\frac{\mu_0 K}{2c}(ct - z)\hat{j}$

Ans. : (d)

Solution: $\vec{B} = \vec{\nabla} \times \vec{A} = \frac{\partial A_x}{\partial z}\hat{y} = -\frac{\mu_0 K}{2c}(ct - z)\hat{j}$

Q25. When a charged particle emits electromagnetic radiation, the electric field $\vec{E}$ and the Poynting vector $\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$ at a larger distance r from emitter vary as $\frac{1}{r^n}$ and $\frac{1}{r^m}$ respectively. Which of the following choices for n and m are correct?

(a) $n = 1$ and $m = 1$ (b) $n = 2$ and $m = 2$

(c) $n = 1$ and $m = 2$ (d) $n = 2$ and $m = 4$

Ans. : (c)

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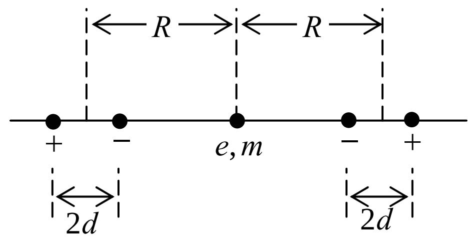
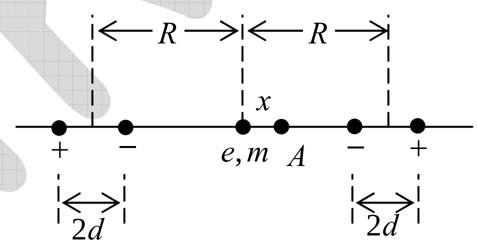
Q26. A particle of charge e and mass m is located at the midpoint of the line joining two fixed collinear dipoles with unit charges as shown in the figure. (The particle is constrained to move only along the line joining the dipoles). Assuming that the length of the dipoles is much shorter than their separation, the natural frequency of oscillation of the particle is



(a) $\sqrt{\dfrac{6eR^2}{\pi\varepsilon_0 md^5}}$ (b) $\sqrt{\dfrac{6eR}{\pi\varepsilon_0 md^4}}$ (c) $\sqrt{\dfrac{6ed^2}{\pi\varepsilon_0 mR^5}}$ (d) $\sqrt{\dfrac{6ed}{\pi\varepsilon_0 mR^4}}$

Ans. : (d)

Solution: Let us displace the charge particle by small amount x at A. Then the resultant electric field at point A is given by



$$E = \frac{2p}{4\pi\varepsilon_0}\left[\frac{1}{(R+x)^3} - \frac{1}{(R-x)^3}\right] = -\frac{6d}{\pi\varepsilon_0 R^4}x,$$

$$F = eE = -\frac{6ed}{\pi\varepsilon_0 R^4}x. \text{ Then, } \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6ed}{\pi\varepsilon_0 mR^4}}$$ (where $p = 1\times 2d = 2d$)

Q27. A current I is created by a narrow beam of protons moving in vacuum with constant velocity $\vec{u}$. The direction and magnitude, respectively of the Poynting vector $\vec{S}$ outside the beam at a radial distance r (much larger than the width of the beam) from the axis, are

(a) $\vec{S} \perp \vec{u}$ and $|\vec{S}| = \dfrac{I^2}{4\pi^2\varepsilon_0|\vec{u}|r^2}$ (b) $\vec{S} \parallel (-\vec{u})$ and $|\vec{S}| = \dfrac{I^2}{4\pi^2\varepsilon_0|\vec{u}|r^4}$

(c) $\vec{S} \parallel \vec{u}$ and $|\vec{S}| = \dfrac{I^2}{4\pi^2\varepsilon_0|\vec{u}|r^2}$ (d) $\vec{S} \parallel \vec{u}$ and $|\vec{S}| = \dfrac{I^2}{4\pi^2\varepsilon_0|\vec{u}|r^4}$

Ans. : (c)

Solution: Let charge per unit length be λ, hence $I = \lambda u$ in z-direction.

The magnetic field at a distance r is $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi}$.

The electric field at a distance r is $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r} = \frac{I}{2\pi\varepsilon_0 u r}\hat{r}$.

Hence Poynting vector $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0} = \frac{I^2}{4\pi^2\varepsilon_0 u r^2}\hat{z}$

Q28. If the electric and magnetic fields are unchanged when the potential $\vec{A}$ changes (in suitable units) according to $\vec{A} \to \vec{A} + \hat{r}$, where $\vec{r} = r(t)\hat{r}$, then the scalar potential Φ must simultaneously change to

(a) $\Phi - r$ (b) $\Phi + r$ (c) $\Phi - \partial r/\partial t$ (d) $\Phi + \partial r/\partial t$

Ans. : (c)

Solution: $\vec{A'} = \vec{A} + \vec{\nabla}\lambda = \vec{A} + \hat{r} \Rightarrow \partial\lambda/\partial r = 1 \Rightarrow \lambda = r + C$

$V' = V - \partial\lambda/\partial t = V - \partial r/\partial t$

Q29. Consider an axially symmetric static charge distribution of the form,

$$\rho = \rho_0\left(\frac{r_0}{r}\right)^2 e^{-r/r_0}\cos^2\varphi$$

The radial component of the dipole moment due to this charge distribution is

(a) $2\pi\rho_0 r_0^4$ (b) $\pi\rho_0 r_0^4$ (c) $\rho_0 r_0^4$ (d) $\pi\rho_0 r_0^4/2$

Ans. : (a)

Solution: $p = \int_V r'\rho(r')d\tau' = \int\int\int r' \times \rho_0\left(\frac{r_0}{r'}\right)^2 e^{-r'/r_0}\cos^2\varphi \times r'^2 \sin\theta dr' d\theta d\varphi$

$$p = \rho_0 r_0^2 \int_{r'=0}^{\infty} r' e^{-r'/r_0} dr' \int_0^{\pi}\sin\theta d\theta \int_0^{2\pi}\cos^2\varphi d\varphi = 2\pi\rho_0 r_0^4$$

Q30. The components of a vector potential $A_\mu \equiv (A_0, A_1, A_2, A_3)$ are given by

$$A_\mu = k(-xyz, yzt, zxt, xyt)$$

where k is a constant. The three components of the electric field are

(a) $k(yz, zx, xy)$ (b) $k(x, y, z)$ (c) $(0,0,0)$ (d) $k(xt, yt, zt)$

Ans. : (c)

Solution: $A_\mu = (\phi, A_x, A_y, A_z) \Rightarrow \phi = -kxyz, A_x = kyzt, A_y = kzxt, A_z = kxyt$

Since $\vec{\nabla}\phi = k(-yz\hat{x} - xz\hat{y} - xy\hat{z})$ and $\frac{\partial \vec{A}}{\partial t} = k(yz\hat{x} + xz\hat{y} + xy\hat{z})$

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t} = k(yz\hat{x} + xz\hat{y} + xy\hat{z}) - k(yz\hat{x} + xz\hat{y} + xy\hat{z}) = 0 \Rightarrow \vec{E} = (0,0,0)$$

Q31. An oscillating current $I(t) = I_0 \exp(-i\omega t)$ flows in the direction of the y-axis through a thin metal sheet of area $1.0\,cm^2$ kept in the xy-plane. The rate of total energy radiated per unit area from the surfaces of the metal sheet at a distance of $100\,m$ is

(a) $I_0\omega/(12\pi\varepsilon_0 c^3)$ (b) $I_0^2\omega^2/(12\pi\varepsilon_0 c^3)$

(c) $I_0^2\omega^3/(12\pi\varepsilon_0 c^3)$ (d) $I_0^2\omega^4/(12\pi\varepsilon_0 c^3)$

Ans. : (d)

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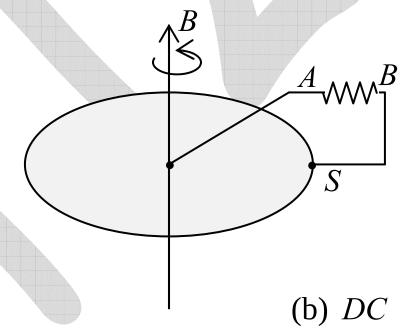
Q32. A horizontal metal disc rotates about the vertical axis in a uniform magnetic field pointing up as shown in the figure. A circuit is made by connecting one end A of a resistor to the centre of the disc and the other end B to its edge through a sliding contact. The current that flows through the resistor is



(a) zero (b) DC from A to B

(c) DC from B to A (d) AC

Ans. : (b)

Q33. The force between two long and parallel wires carrying currents I_1 and I_2 and separated by a distance D is proportional to

(a) $I_1 I_2 / D$ (b) $(I_1 + I_2)/D$ (c) $(I_1 I_2 / D)^2$ (d) $I_1 I_2 / D^2$

Ans. : (a)

Q34. The electric field of an electromagnetic wave is given by

$$\vec{E} = E_0 \cos[\pi(0.3x + 0.4y - 1000t)]\hat{k}.$$

The associated magnetic field $\vec{B}$ is

(a) $10^{-3} E_0 \cos[\pi(0.3x + 0.4y - 1000t)]\hat{k}$

(b) $10^{-4} E_0 \cos[\pi(0.3x + 0.4y - 1000t)](4\hat{i} - 3\hat{j})$

(c) $E_0 \cos[\pi(0.3x + 0.4y - 1000t)](0.3\hat{i} + 0.4\hat{j})$

(d) $10^{2} E_0 \cos[\pi(0.3x + 0.4y - 1000t)](3\hat{i} + 4\hat{j})$

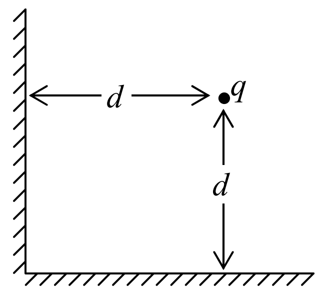
Ans. : (b)

Solution: $\vec{k} = \pi(0.3\hat{x} + 0.4\hat{y}), \omega = 1000\pi$

$$\vec{B} = \frac{\vec{k} \times \vec{E}}{\omega} = \frac{1}{\omega}\pi(0.3\hat{x} + 0.4\hat{y}) \times E_0 \cos[\pi(0.3x + 0.4y - 1000t)]\hat{k}$$

$$\Rightarrow \vec{B} = 10^{-4} E_0 \cos[\pi(0.3x + 0.4y - 1000t)](4\hat{i} - 3\hat{j})$$

Q35. A point charge q is placed symmetrically at a distance d from two perpendicularly placed grounded conducting infinite plates as shown in the figure. The net force on the charge (in units of $1/4\pi\varepsilon_0$) is



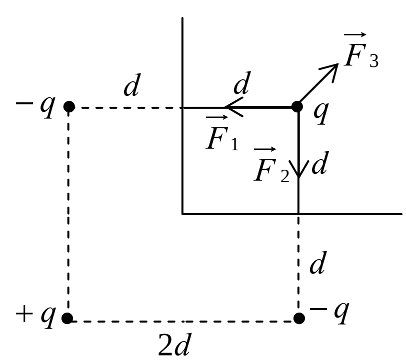
(a) $\frac{q^2}{8d^2}(2\sqrt{2} - 1)$ away from the corner

(b) $\frac{q^2}{8d^2}(2\sqrt{2} - 1)$ towards the corner

(c) $\frac{q^2}{2\sqrt{2}d^2}$ towards the corner

(d) $\frac{3q^2}{8d^2}$ away from the corner

Ans. : (b)



Solution: $\left|\vec{F}_1\right| = \left|\vec{F}_2\right| = k\frac{q^2}{4d^2}$ and $\left|\vec{F}_3\right| = k\frac{q^2}{8d^2}$

Resultant of $\vec{F}_1, \vec{F}_2$ is $F_{12} = \sqrt{F_1^2 + F_2^2} = 2\sqrt{2}k\frac{q^2}{8d^2}$.

Net force $\vec{F} = k\dfrac{q^2}{8d^2}\left(2\sqrt{2}-1\right)$(towards the corner)

Q36. If the electrostatic potential $V(r,\theta,\phi)$ in a charge free region has the form $V(r,\theta,\phi) = f(r)\cos\theta$, then the functional form of $f(r)$ (in the following a and b are constants) is:

(a) $ar^2 + \dfrac{b}{r}$ (b) $ar + \dfrac{b}{r^2}$ (c) $ar + \dfrac{b}{r}$ (d) $a\ln\left(\dfrac{r}{b}\right)$

Ans. : (b)

Solution: $\nabla^2 V = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial V}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial V}{\partial\theta}\right) + \dfrac{1}{r^2\sin^2\theta}\left(\dfrac{\partial^2 V}{\partial\phi^2}\right) = 0$

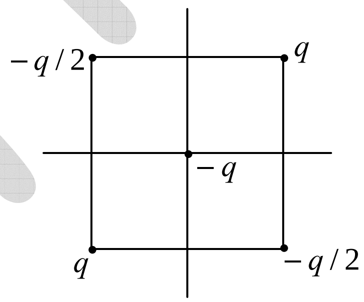
$$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\cos\theta\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left[\sin\theta f\times(-\sin\theta)\right] = 0$$

$$\Rightarrow \frac{\cos\theta}{r^2}\left(r^2\frac{\partial^2 f}{\partial^2 r} + 2r\frac{\partial f}{\partial r}\right) - \frac{f}{r^2\sin\theta}\left(2\sin\theta\cos\theta\right) = 0$$

$$\Rightarrow r^2\frac{\partial^2 f}{\partial^2 r} + 2r\frac{\partial f}{\partial r} - 2f(r) = 0$$

$f(r) = ar + \dfrac{b}{r^2}$ satisfy the above equation.

Q37. Let four point charges q, $-q/2$, q and $-q/2$ be placed at the vertices of a square of side a. Let another point charge $-q$ be placed at the centre of the square (see the figure).



Let $V(r)$ be the electrostatic potential at a point P at a distance $r >> a$ from the centre of the square. Then $V(2r)/V(r)$ is

(a) 1 (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{4}$ (d) $\dfrac{1}{8}$

Ans. : (d)

Solution: According to multipole expansion $Q_{mono}=-\frac{q}{2}+q-\frac{q}{2}+q-q=0$

$$\vec{p}=q\left(\frac{q}{2}\hat{x}+\frac{q}{2}\hat{y}\right)-\frac{q}{2}\left(-\frac{q}{2}\hat{x}+\frac{q}{2}\hat{y}\right)+0+q\left(-\frac{q}{2}\hat{x}-\frac{q}{2}\hat{y}\right)-\frac{q}{2}\left(\frac{q}{2}\hat{x}-\frac{q}{2}\hat{y}\right)=0$$

Thus, $V\propto\frac{1}{r^3}\Rightarrow\frac{V(2r)}{V(r)}=\frac{1}{8}$.

Q38. Let $\left(V,\vec{A}\right)$ and $\left(V',\vec{A'}\right)$ denote two sets of scalar and vector potentials, and ψ is a scalar function. Which of the following transformations leave the electric and magnetic fields (and hence Maxwell's equations) unchanged?

(a) $\vec{A'}=\vec{A}+\nabla\psi$ and $V'=V-\frac{\partial\psi}{\partial t}$

(b) $\vec{A'}=\vec{A}-\nabla\psi$ and $V'=V+2\frac{\partial\psi}{\partial t}$

(c) $\vec{A'}=\vec{A}+\nabla\psi$ and $V'=V+\frac{\partial\psi}{\partial t}$

(d) $\vec{A'}=\vec{A}-\nabla\psi$ and $V'=V-\frac{\partial\psi}{\partial t}$

Ans. : (a)

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Q39. A time-dependent current $\vec{I}(t)=Kt\hat{z}$ (where K is a constant) is switched on at $t=0$ in an infinite current-carrying wire. The magnetic vector potential at a perpendicular distance a from the wire is given (for time $t>a/c$) by

(a) $\hat{z}\frac{\mu_0 K}{4\pi c}\int\limits_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}}dz\frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$

(b) $\hat{z}\frac{\mu_0 K}{4\pi}\int\limits_{-ct}^{ct}dz\frac{t}{\left(a^2+z^2\right)^{1/2}}$

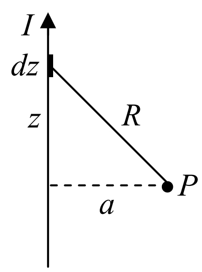
(c) $\hat{z}\frac{\mu_0 K}{4\pi c}\int\limits_{-ct}^{ct}dz\frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$

(d) $\hat{z}\frac{\mu_0 K}{4\pi}\int\limits_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}}dz\frac{t}{\left(a^2+z^2\right)^{1/2}}$

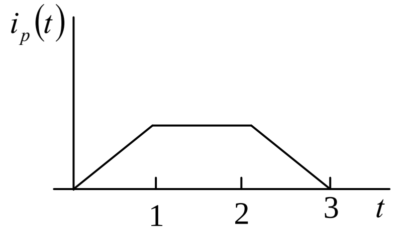
Ans. : (a)

Solution: $\vec{A}=\hat{z}\frac{\mu_0}{4\pi}\int\limits_{-\infty}^{\infty}\frac{I\left(t_r\right)}{R}dz=\hat{z}\frac{\mu_0}{4\pi}\int\limits_{-\infty}^{\infty}\frac{K\left(t-R/c\right)}{R}dz$

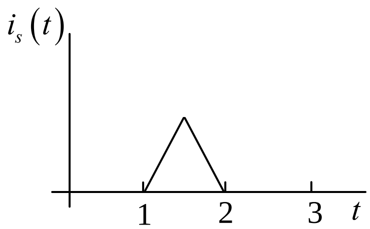
$$\Rightarrow\vec{A}=\hat{z}\frac{\mu_0 K}{4\pi c}\int\limits_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}}dz\frac{ct-\sqrt{a^2+z^2}}{\left(a^2+z^2\right)^{1/2}}$$

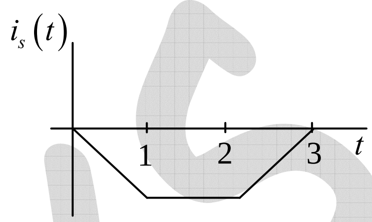


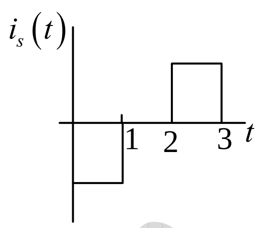
Q40. A current i_p flows through the primary coil of a transformer. The graph of $i_p(t)$ as a function of time t is shown in the figure below.

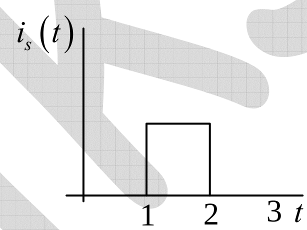


Which of the following graphs represents the current i_S in the secondary coil?

(a)


(b)


(c)


(d)


Ans. : (c)

Solution: $i_s \propto -\dfrac{di_p}{dt}$

Q41. If the electrostatic potential in spherical polar coordinates is

$$\varphi(r) = \varphi_0 e^{-r/r_0}$$

where φ_0 and r_0 are constants, then the charge density at a distance $r = r_0$ will be

(a) $\dfrac{\varepsilon_0 \varphi_0}{e r_0^2}$ (b) $\dfrac{e \varepsilon_0 \varphi_0}{2 r_0^2}$ (c) $-\dfrac{\varepsilon_0 \varphi_0}{e r_0^2}$ (d) $-\dfrac{2e \varepsilon_0 \varphi_0}{r_0^2}$

Ans. : (a)

Solution: $\because \nabla^2 \phi = -\dfrac{\rho}{\varepsilon_0} \Rightarrow \rho = -\varepsilon_0 \left(\nabla^2 \phi\right)$

$$\nabla^2 \phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \phi}{\partial r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \times -\frac{\phi_0}{r_0} e^{-r/r_0}\right) = -\frac{1}{r^2}\frac{\phi_0}{r_0}\frac{\partial}{\partial r}\left(r^2 \times e^{-r/r_0}\right)$$

$$\Rightarrow \nabla^2 \phi = -\frac{1}{r^2}\frac{\phi_0}{r_0}\left[r^2 \times -\frac{1}{r_0} e^{-r/r_0} + 2r e^{-r/r_0}\right] = -\frac{\phi_0}{r_0}\left[-\frac{1}{r_0} e^{-r/r_0} + \frac{2}{r} e^{-r/r_0}\right]$$

At a distance $r = r_0$, $\nabla^2\phi = -\frac{\phi_0}{r_0}\left[\frac{-1}{r_0}e^{-1} + \frac{2}{r_0}e^{-1}\right] = -\frac{\phi_0}{r_0^2 e} \Rightarrow \rho = -\varepsilon_0\left(-\frac{\phi_0}{r_0^2 e}\right) = \frac{\phi_0\varepsilon_0}{r_0^2 e}$

Q42. If $\vec{A} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and C is the circle of unit radius in the plane defined by $z = 1$, with the centre on the z - axis, then the value of the integral $\oint_C \vec{A}\cdot d\vec{\ell}$ is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 0

Ans. : (d)

Solution: $\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix} = \hat{i}(x-x) - \hat{j}(y-y) + \hat{k}(z-z) = 0$

Since $\oint_C \vec{A}\cdot d\vec{\ell} = \int_S \left(\vec{\nabla}\times\vec{A}\right)\cdot d\vec{a} = 0$

Q43. Consider an electromagnetic wave at the interface between two homogenous dielectric media of dielectric constants ε_1 and ε_2. Assuming $\varepsilon_2 > \varepsilon_1$ and no charges on the surface, the electric field vector $\vec{E}$ and the displacement vector $\vec{D}$ in the two media satisfy the following inequalities

(a) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$ (b) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$

(c) $|\vec{E}_2| < |\vec{E}_1|$ and $|\vec{D}_2| > |\vec{D}_1|$ (d) $|\vec{E}_2| > |\vec{E}_1|$ and $|\vec{D}_2| < |\vec{D}_1|$

Ans. : (c)

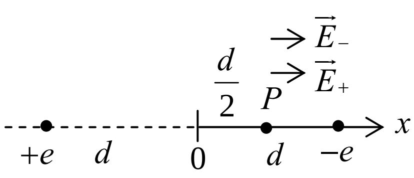
Q44. A charge $(-e)$ is placed in vacuum at the point $(d,0,0)$, where $d > 0$. The region $x \le 0$ is filled uniformly with a metal. The electric field at the point $\left(\frac{d}{2},0,0\right)$ is

(a) $-\frac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$ (b) $\frac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$

(c) $\frac{e}{\pi\varepsilon_0 d^2}(1,0,0)$ (d) $-\frac{e}{\pi\varepsilon_0 d^2}(1,0,0)$

Ans. : (b)

Solution:



$E_+ = \frac{1}{4\pi\varepsilon_0}\frac{e}{(3d/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4e}{9d^2}$ and $E_- = \frac{1}{4\pi\varepsilon_0}\frac{e}{(d/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{4e}{d^2}$

Thus resultant electric field at point P is

$$E = E_+ + E_- = \frac{1}{4\pi\varepsilon_0}\frac{4e}{9d^2} + \frac{1}{4\pi\varepsilon_0}\frac{4e}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{40e}{9d^2} = \frac{1}{9\pi\varepsilon_0}\frac{10e}{d^2} \Rightarrow \vec{E} = \frac{1}{9\pi\varepsilon_0}\frac{10e}{d^2}\hat{x}$$

Q45. A beam of light of frequency ω is reflected from a dielectric-metal interface at normal incidence. The refractive index of the dielectric medium is n and that of the metal is $n_2 = n(1+i\rho)$. If the beam is polarised parallel to the interface, then the phase change experienced by the light upon reflection is

(a) $\tan(2/\rho)$ (b) $\tan^{-1}(1/\rho)$ (c) $\tan^{-1}(2/\rho)$ (d) $\tan^{-1}(2\rho)$

Ans. : (c)

Solution: Since $\tilde{E}_{0R} = \left(\frac{1-\tilde{\beta}}{1+\tilde{\beta}}\right)\tilde{E}_{0I}$ where $\tilde{\beta} = \frac{v_1}{v_2} = \frac{c/n}{c/n(1+i\rho)} = 1+i\rho$

$$\Rightarrow \tilde{E}_{0R} = \left(\frac{-i\rho}{2+i\rho}\right)\tilde{E}_{0I} = \left(\frac{\rho e^{-i\pi/2}}{\sqrt{4+\rho^2}e^{i\theta}}\right)\tilde{E}_{0I} = \left(\frac{\rho}{\sqrt{4+\rho^2}}\right)e^{-i(\pi/2+\theta)}\tilde{E}_{0I} \text{ where } \tan\theta = \frac{\rho}{2}.$$

Thus phase change $\phi = -(\pi/2+\theta) \Rightarrow \tan\phi = \cot\theta = \frac{2}{\rho} \Rightarrow \phi = \tan^{-1}\left(\frac{2}{\rho}\right)$

Q46. A thin, infinitely long solenoid placed along the z - axis contains a magnetic flux ϕ. Which of the following vector potentials corresponds to the magnetic field at an arbitrary point (x, y, z)?

(a) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2}, \frac{\phi}{2\pi}\frac{x}{x^2+y^2}, 0\right)$

(b) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{y}{x^2+y^2+z^2}, \frac{\phi}{2\pi}\frac{x}{x^2+y^2+z^2}, 0\right)$

(c) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{x+y}{x^2+y^2}, \frac{\phi}{2\pi}\frac{x+y}{x^2+y^2}, 0\right)$

(d) $(A_x, A_y, A_z) = \left(-\frac{\phi}{2\pi}\frac{x}{x^2+y^2}, \frac{\phi}{2\pi}\frac{y}{x^2+y^2}, 0\right)$

Ans. : (a)

Solution: $\vec{B} = \vec{\nabla}\times\vec{A} = 0$

Q47. An electromagnetically-shielded room is designed so that at a frequency $\omega = 10^7$ rad/s the intensity of the external radiation that penetrates the room is 1% of the incident radiation. If $\sigma = \frac{1}{2\pi}\times 10^6 (\Omega m)^{-1}$ is the conductivity of the shielding material, its minimum thickness should be (given that $\ln 10 = 2.3$)

(a) 4.60 mm (b) 2.30 mm (c) 0.23 mm (d) 0.46 mm

Ans. : (b)

Solution: $I = I_0 e^{-2\kappa z} \Rightarrow z = \frac{1}{2\kappa}\ln\left(\frac{I_0}{I}\right)$

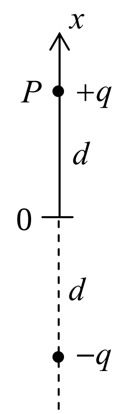
where $\frac{I_0}{I} = 100,\ \kappa = \sqrt{\frac{\sigma\mu\omega}{2}} = \sqrt{\frac{1}{2}\times\frac{1}{2\pi}\times 10^6 \times 4\pi\times 10^{-7}\times 10^7} = 10^3$

$\Rightarrow z = \frac{1}{2\times 10^3}\ln(100) = 2.30\, mm$

Q48. A charged particle is at a distance d from an infinite conducting plane maintained at zero potential. When released from rest, the particle reaches a speed u at a distance $d/2$ from the plane. At what distance from the plane will the particle reach the speed $2u$?

(a) $d/6$ (b) $d/3$ (c) $d/4$ (d) $d/5$

Ans. : (d)



Solution: $F = ma = m\frac{d^2x}{dt^2} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{4d^2} \Rightarrow \frac{d^2x}{dt^2} = -\frac{A}{x^2}$ where $A = \frac{q^2}{16\pi m\varepsilon_0}$.

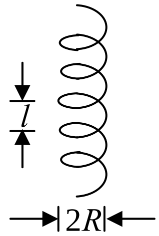
$\Rightarrow \frac{dv}{dt} = -\frac{A}{x^2} \Rightarrow v\frac{dv}{dt} = -\frac{A}{x^2}\frac{dx}{dt} \Rightarrow \frac{1}{2}\frac{d}{dt}\left(v^2\right) = \frac{d}{dt}\left(\frac{A}{x}\right)$

$\Rightarrow \frac{v^2}{2} = \frac{A}{x} + C$ at $\Rightarrow x = d,\ v = 0 \Rightarrow C = -\frac{A}{d} \Rightarrow v = \sqrt{2A}\sqrt{\left(\frac{1}{x}-\frac{1}{d}\right)}$.

Thus $u = \sqrt{2A}\sqrt{\left(\frac{1}{d/2}-\frac{1}{d}\right)} = \sqrt{\frac{2A}{d}}$ then $2u = \sqrt{2A}\sqrt{\left(\frac{1}{x}-\frac{1}{d}\right)} \Rightarrow x = \frac{d}{5}$

NET/JRF–(DEC-2014)

Q49. A charged particle moves in a helical path under the influence of a constant magnetic field. The initial velocity is such that the component along the magnetic field is twice the component in the plane normal to the magnetic field.



The ratio ℓ / R of the pitch ℓ to the radius R of the helical path is

(a) $\pi / 2$ (b) 4π (c) 2π (d) π

Ans. : (b)

Solution: $v_{\|} = 2v_{\perp}$

Pitch of the helix $l = v_{\|} T = v_{\|} \dfrac{2\pi R}{v_{\perp}} = 2v_{\perp} \dfrac{2\pi R}{v_{\perp}} = 4\pi R \Rightarrow \dfrac{l}{R} = 4\pi$

Q50. A parallel beam of light of wavelength λ is incident normally on a thin polymer film with air on both sides. If the film has a refractive index $n > 1$, then second-order bright fringes can be observed in reflection when the thickness of the film is

(a) $\lambda / 4n$ (b) $\lambda / 2n$ (c) $3\lambda / 4n$ (d) λ / n

Ans. : (c)

Solution: For constructive interference: $2nd \cos\theta = (2m+1)\dfrac{\lambda}{2}$

For normal incidence $(\theta = 0)$ and second order $(m = 1)$

$\Rightarrow 2nd \cos 0 = (2 \times 1 + 1)\dfrac{\lambda}{2} \Rightarrow d = \dfrac{3\lambda}{4n}$

Q51. A solid sphere of radius R has a charge density, given by

$$\rho(r) = \rho_0 \left(1 - \frac{ar}{R}\right)$$

where r is the radial coordinate and ρ_0, a and R are positive constants. If the magnitude of the electric field at $r = R/2$ is 1.25 times that at $r = R$, then the value of a is

(a) 2 (b) 1 (c) $1/2$ (d) $1/4$

Ans. : (b)

Solution: $\oint_S \vec{E}.d\vec{a} = \frac{1}{\varepsilon_0}Q_{enc} \Rightarrow \left|\vec{E}\right| \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r \rho_0\left(1-\frac{ar}{R}\right)4\pi r^2 dr$

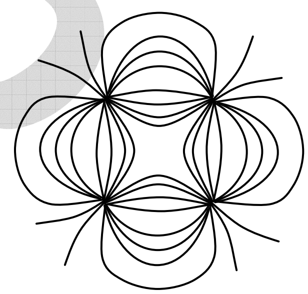
$\Rightarrow \left|\vec{E}\right| \times 4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\int_0^r \left(r^2 - \frac{ar^3}{R}\right)dr = \frac{4\pi\rho_0}{\varepsilon_0}\left(\frac{r^3}{3}-\frac{ar^4}{4R}\right) \Rightarrow \left|\vec{E}\right| = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3}-\frac{ar^2}{4R}\right)$

$\because E_{r=R/2} = 1.25E_{r=R} \Rightarrow \frac{\rho_0}{\varepsilon_0}\left(\frac{R/2}{3}-\frac{aR^2/4}{4R}\right) = 1.25\frac{\rho_0}{\varepsilon_0}\left(\frac{R}{3}-\frac{aR^2}{4R}\right)$

$\Rightarrow \left(\frac{1}{6}-\frac{a}{16}\right) = \frac{5}{4}\left(\frac{1}{3}-\frac{a}{4}\right) \Rightarrow \left(\frac{1}{6}-\frac{a}{16}\right) = \left(\frac{5}{12}-\frac{5a}{16}\right) \Rightarrow \frac{5a}{16}-\frac{a}{16} = \frac{5}{12}-\frac{1}{6}$

$\Rightarrow \frac{4a}{16} = \frac{5-2}{12} \Rightarrow \frac{a}{4} = \frac{3}{12} \Rightarrow a = 1$

Q52. The electrostatic lines of force due to a system of four point charges is sketched here. At large distance r, the leading asymptotic behaviour of the electrostatic potential is proportional to



(a) r (b) r^{-1}

(c) r^{-2} (d) r^{-3}

Ans. : (d)

Solution: The given electrostatic line of force is due to a quadrupole. So $V \propto \frac{1}{r^3}$.

Q53. A plane electromagnetic wave incident normally on the surface of a material is partially reflected. Measurements on the standing wave in the region in front of the interface such that the ratio of the electric field amplitude at the maxima and the minima is 5. The ratio of the reflected intensity to the incident intensity is

(a) $4/9$ (b) $2/3$ (c) $2/5$ (d) $1/5$

Ans. : (a)

Solution: $\frac{E_{0I}+E_{0R}}{E_{0I}-E_{0R}} = 5 \Rightarrow E_{0I}+E_{0R} = 5\left(E_{0I}-E_{0R}\right) \Rightarrow 6E_{0R} = 4E_{0I} \Rightarrow \frac{E_{0R}}{E_{0I}} = \frac{2}{3}$

$\Rightarrow \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \frac{4}{9}$

Q54. A non-relativistic particle of mass m and charge e, moving with a velocity $\vec{v}$ and acceleration $\vec{a}$, emits radiation of intensity I. What is the intensity of the radiation emitted by a particle of mass $m/2$, charge $2e$, velocity $\vec{v}/2$ and acceleration $2\vec{a}$?

(a) 16 I (b) 8 I (c) 4 I (d) 2 I

Ans. : (a)

Solution: $\because I \propto \dfrac{q^2a^2\sin^2\theta}{r^2} \Rightarrow \dfrac{I_2}{I_1} = \dfrac{q_2^2a_2^2}{q_1^2a_1^2} \Rightarrow \dfrac{I_2}{I} = \dfrac{4e^2 \times 4a^2}{e^2a^2} = 16 \Rightarrow I_2 = 16I$

NET/JRF–(JUNE-2015)

Q55. A Plane electromagnetic wave is travelling along the positive z-direction. The maximum electric field along the x- direction is 10 V/m. The approximate maximum values of the power per unit area and the magnetic induction B, respectively, are

(a) $3.3\times10^{-7}\, watts/m^2$ and 10 tesla

(b) $3.3\times10^{-7}\, watts/m^2$ and 3.3×10^{-8} tesla

(c) 0.265 $watts/m^2$ and 10 tesla

(d) 0.265 $watts/m^2$ and 3.3×10^{-8} tesla

Ans. (d)

Solution: $E_0 = 10V/m, I = \dfrac{P}{A} = \dfrac{1}{2}c\in_0 E_0^2 = \dfrac{1}{2}\times3\times10^8\times8.86\times10^{-12}\times(10)^2 = 0.132\ W/m^2$

$B_0 = \dfrac{E_0}{c} = \dfrac{10}{3\times10^8} = 3.3\times10^{-8}$ Tesla

Q56. Which of the following transformations $(V,\vec{A}) \to (V',\vec{A}')$ of the electrostatic potential V and the vector potential $\vec{A}$ is a gauge transformation?

(a) $\left(V' = V + ax, \vec{A}' = \vec{A} + at\ \hat{k}\right)$ (b) $\left(V' = V + ax, \vec{A}' = \vec{A} - at\ \hat{k}\right)$

(c) $\left(V' = V + ax, \vec{A}' = \vec{A} + at\ \hat{i}\right)$ (d) $\left(V' = V + ax, \vec{A}' = \vec{A} - at\ \hat{i}\right)$

Ans. (d)

Solution: $V' = V - \dfrac{\partial\lambda}{\partial t} \Rightarrow \dfrac{\partial\lambda}{\partial t} = -ax \Rightarrow \lambda = -axt + c$

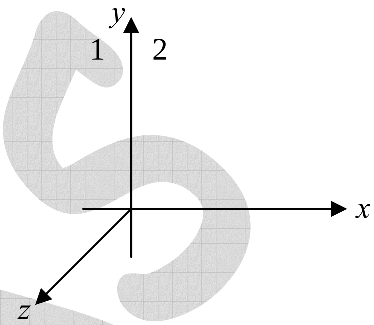
$\Rightarrow \vec{\Delta}\lambda + at\hat{i} = 0$. Thus, $\vec{A} = \vec{A} - at\hat{x}$

Q57. Suppose the yz-plane forms a chargeless boundary between two media of permittivities $\in_{\text{left}}$ and $\in_{\text{right}}$ where $\in_{\text{left}} : \in_{\text{right}} = 1:2$, if the uniform electric field on the left is $\vec{E}_{\text{left}} = c\left(\hat{i}+\hat{j}+\hat{k}\right)$ (where c is a constant), then the electric field on the right $\vec{E}_{\text{right}}$ is

(a) $c\left(2\hat{i}+\hat{j}+\hat{k}\right)$ (b) $c\left(\hat{i}+2\hat{j}+2\hat{k}\right)$

(c) $c\left(\frac{1}{2}\hat{i}+\hat{j}+\hat{k}\right)$ (d) $c\left(\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}\right)$

Ans. (c)



Solution: $E_1^{\parallel} = c\left(\hat{j}+\hat{k}\right) = E_2^{\parallel}$

$$D_1^{\perp} = D_2^{\perp} \Rightarrow \in_1 E_1^{\perp} = \in_2 E_2^{\perp} \Rightarrow E_2^{1} = \frac{\in_1}{\in_2} E_1^{\perp}$$

$$\Rightarrow E_2^{\perp} = \frac{1}{2}c\hat{i} \Rightarrow \vec{E}_2 = c\left(\frac{1}{2}\hat{i}+\hat{j}+\hat{k}\right)$$

Q58. A proton moves with a speed of $300\ m/s$ in a circular orbit in the xy-plan in a magnetic field 1 tesla along the positive z- direction. When an electric field of $1\ V/m$ is applied along the positive y-direction, the center of the circular orbit

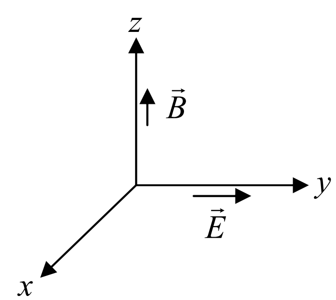
(a) remains stationary

(b) moves at $1\ m/s$ along the negative $x-$ direction

(c) moves at $1\ m/s$ along the positive $z-$ direction

(d) moves at $1\ m/s$ along the positive $x-$ direction

Ans. (d)



Solution: Change particle will deflect in $+x$-direction with

$$v = \frac{E}{B} = \frac{1}{1} = 1\ m/s.$$

Q59. Consider a rectangular wave guide with transverse dimensions $2\ m \times 1\ m$ driven with an angular frequency $\omega = 10^9\ rad/s$. Which transverse electric (TE) modes will propagate in this wave guide?

(a) TE_{10}, TE_{01} and TE_{20} (b) TE_{01}, TE_{11} and TE_{20}

(c) TE_{01}, TE_{10} and TE_{11} (d) TE_{01}, TE_{10} and TE_{22}

Ans. (a)

Solution: $\omega_{mn} = C\pi\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}}$

$$\omega_{10} = \frac{c\pi}{a} = \frac{3\times10^8\times3.14}{2} = 4.71\times10^8 \; rod/\sec$$

$$\omega_{01} = \frac{c\pi}{b} = \frac{3\times10^8\times3.14}{1} = 9.42\times10^8 \; rod/\sec$$

$$\omega_{11} = c\pi\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = 10.53\times10^8 \; rod/\sec$$

$$\omega_{20} = \frac{2c\pi}{a} = 9.72\times10^8 \; rod/\sec$$

$$\omega_{22} = c\pi\sqrt{\frac{4}{a^2}+\frac{4}{b^2}} = 10.5\times10^8 \; rod/\sec$$

Since $\omega > \omega_{10}, \omega_{01}, \omega_{20}$

Q60. The electric and magnetic fields in the charge free region $z > 0$ are given by

$$\vec{E}(\vec{r},t) = E_0 e^{-k_1 z}\cos(k_2 x - \omega t)\hat{j}$$

$$\vec{B}(\vec{r},t) = \frac{E_0}{\omega}e^{-k_1 z}\left[k_1\sin(k_2 x-\omega t)\hat{i} + k_2\cos(k_2 x - \omega t)\hat{k}\right]$$

where ω, k_1 and k_2 are positive constants. The average energy flow in the x-direction is

(a) $\frac{E_0^2 k_2}{2\mu_0\omega}e^{-2k_1 z}$ (b) $\frac{E_0^2 k_2}{\mu_0\omega}e^{-2k_1 z}$ (c) $\frac{E_0^2 k_1}{2\mu_0\omega}e^{-2k_1 z}$ (d) $\frac{1}{2}c \in_0 E_0^2 e^{-2k_1 z}$

Ans. (a)

Solution: $\vec{S} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right) = \frac{E_0^2 e^{-2k_1 z}}{\mu_0\omega}\left[k_1\cos\theta\sin\theta\left(-\hat{k}\right) + k_2\cos^2\theta\hat{i}\right]$, where $\theta = k_2 x - \omega t$

$$\Rightarrow \quad \langle\vec{S}\rangle = \frac{k_2}{2}\frac{E_0^2 e^{-2k_1 z}}{\mu_0\omega} = \frac{E_0^2 k_2}{2\mu_0\omega}e^{-2k_1 z}$$

Q61. A uniform magnetic field in the positive z-direction passes through a circular wire loop of radius $1\,cm$ and resistance $1\,\Omega$ lying in the xy-plane. The field strength is reduced from 10 tesla to 9 tesla in $1s$. The charge transferred across any point in the wire is approximately

(a) 3.1×10^{-4} coulomb (b) 3.4×10^{-4} coulomb

(c) 4.2×10^{-4} coulomb (d) 5.2×10^{-4} coulomb

Ans. (a)

Solution: $\varepsilon = -\frac{d\phi}{dt} \Rightarrow I = \frac{dq}{dt} = \frac{\varepsilon}{R} = -\frac{1}{R}\frac{d\phi}{dt} \Rightarrow \; dq = -\frac{A}{R}dB = \frac{-\pi r^2}{R}dB$

$$\Rightarrow \; dq = \frac{-3.14\times\left(10^{-2}\right)^2}{1}\times 1 = 3.14\times 10^{-4} \text{ coulomb}$$

Q62. A rod of length L carries a total charge Q distributed uniformly. If this is observed in a frame moving with a speed v along the rod, the charge per unit length (as measured by the moving observer) is

(a) $\frac{Q}{L}\left(1-\frac{v^2}{c^2}\right)$ (b) $\frac{Q}{L}\sqrt{1-\frac{v^2}{c^2}}$ (c) $\frac{Q}{L\sqrt{1-\frac{v^2}{c^2}}}$ (d) $\frac{Q}{L\left(1-\frac{v^2}{c^2}\right)}$

Ans. : (c)

Solution: $\lambda = \frac{\lambda_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{Q}{L\sqrt{1-\frac{v^2}{c^2}}}$

NET/JRF–(DEC-2015)

Q63. A hollow metallic sphere of radius a, which is kept at a potential V_0 has a charge Q at its centre. The potential at a point outside the sphere, at a distance r from the centre, is

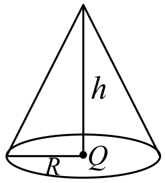
(a) V_0 (b) $\frac{Q}{4\pi\in_0 r}+\frac{V_0 a}{r}$ (c) $\frac{Q}{4\pi\in_0 r}+\frac{V_0 a^2}{r^2}$ (d) $\frac{V_0 a}{r}$

Ans. : (d)

Solution: Let charge on conductor is Q, then $V_0 = \frac{Q}{4\pi\in_0 a}$

Now $V = \frac{Q}{4\pi\in_0 r} \Rightarrow V = \frac{V_0 a}{r}$

Q64. Consider a charge Q at the origin of 3- dimensional coordinate system. The flux of the electric field through the curved surface of a cone that has a height h and a circular base of radius R (as shown in the figure) is



(a) $\frac{Q}{\in_0}$ (b) $\frac{Q}{2\in_0}$ (c) $\frac{hQ}{R\in_0}$ (d) $\frac{QR}{2h\in_0}$

Ans. : (b)

Q65. Given a uniform magnetic field $B = B_0\hat{k}$ (where B_0 is a constant), a possible choice for the magnetic vector potential A is

(a) $B_0 y\hat{i}$ (b) $-B_0 y\hat{i}$ (c) $B_0\left(x\hat{j}+y\hat{i}\right)$ (d) $B_0\left(x\hat{i}+y\hat{j}\right)$

Ans. : (b)

Solution: (a) $\vec{\nabla}\times\vec{A} = -B_0\hat{k}$

(b) $\vec{\nabla}\times\vec{A} = B_0\hat{k}$

(c) $\vec{\nabla}\times\vec{A} = 0$

(d) $\vec{\nabla}\times\vec{A} = 0$

Q66. A beam of unpolarized light in a medium with dielectric constant ϵ_1 is reflected from a plane interface formed with another medium of dielectric constant $\epsilon_2 = 3\epsilon_1$. The two media have identical magnetic permeability. If the angle of incidence is 60^0, then the reflected light

(a) is plane polarized perpendicular to the plane of incidence

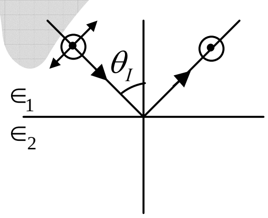
(b) is plane polarized parallel to the plane of incidence

(c) is circularly polarized

(d) has the same polarization as the incident light

Ans. : (a)

Solution: $\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$



$\theta_B = \tan^{-1}\left(\frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1}}\right) = \tan^{-1}\left(\sqrt{3}\right)$

$\Rightarrow \theta_B = 60^0$ (hence reflected light is plane polarized perpendicular to plane of incidence))

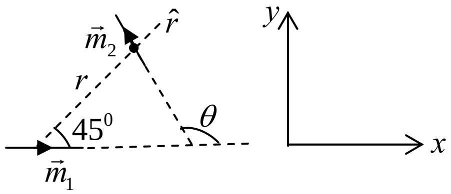
Q67. A small magnetic needle is kept at $(0,0)$ with its moment along the x-axis. Another small magnetic needle is at the point $(1,1)$ and is free to rotate in the xy- plane. In equilibrium the angle θ between their magnetic moments is such that

(a) $\tan\theta = \frac{1}{3}$ (b) $\tan\theta = 0$ (c) $\tan\theta = 3$ (d) $\tan\theta = 1$

Ans. : (c)

Solution: $U = \frac{\mu_0}{4\pi r^3}\left[\vec{m}_1 \cdot \vec{m}_2 - 3\left(\vec{m}_1 \cdot \hat{r}\right)\left(\vec{m}_2 \cdot \hat{r}\right)\right] \Rightarrow U = \frac{\mu_0 m_1 m_2}{4\pi r^3}\left[\cos\theta - 3\cos 45^0 \cos\left(\theta - 45^0\right)\right]$

For stable position energy is minimum i.e.



$$\frac{\partial U}{\partial \theta} = 0 \Rightarrow \frac{\mu_0 m_1 m_2}{4\pi r^3}\left[-\sin\theta + \frac{3}{\sqrt{2}}\sin\left(\theta - 45^0\right)\right] = 0$$

$$\Rightarrow \sin\theta = \frac{3}{\sqrt{2}}\left(\frac{\sin\theta}{\sqrt{2}} - \frac{\cos\theta}{\sqrt{2}}\right) \Rightarrow \tan\theta = 3$$

so, option (c) is correct .

Q68. A dipole of moment $\vec{p}$, oscillating at frequency ω, radiates spherical waves. The vector potential at large distance is

$$\vec{A}\left(\vec{r}\right) = \frac{\mu_0}{4\pi} i\omega \frac{e^{ikr}}{r}\vec{p}$$

To order $\left(\frac{1}{r}\right)$ the magnetic field $\vec{B}$ at a point $\vec{r} = r\hat{n}$ is

(a) $-\frac{\mu_0}{4\pi}\frac{\omega^2}{C}\left(\hat{n}\cdot\vec{p}\right)\hat{n}\frac{e^{ikr}}{r}$ (b) $-\frac{\mu_0}{4\pi}\frac{\omega^2}{C}\left(\hat{n}\times\vec{p}\right)\frac{e^{ikr}}{r}$

(c) $-\frac{\mu_0}{4\pi}\omega^2 k\left(\hat{n}\cdot\vec{p}\right)\vec{p}\frac{e^{ikr}}{r}$ (d) $-\frac{\pi_0}{4\pi}\frac{\omega^2}{C}\vec{p}\frac{e^{ikr}}{r}$

Ans. : (b)

Solution: Let $\vec{p} = p\hat{z}$, then $\vec{B}$ must be in $\hat{\phi}$ direction.

Check $\hat{n}\times\vec{p} = \hat{r}\times\hat{z} = \hat{\phi}$. So, correct option is (b).

Q69. The frequency dependent dielectric constant of a material is given by

$$\varepsilon\left(\omega\right) = 1 + \frac{A}{\omega_0^2 - \omega^2 - i\omega\gamma}$$

where A is a positive constant, ω_0 is the resonant frequency and γ is the damping coefficient. For an electromagnetic wave of angular frequency $\omega << \omega_0$, which of the following is true? (Assume that $\frac{\gamma}{\omega_0} << 1$).

(a) There is negligible absorption of the wave

(b) The wave propagation is highly dispersive

(c) There is strong absorption of the electromagnetic wave

(d) The group velocity and the phase velocity will have opposite sign

Ans. : (a)

Solution: When $\omega << \omega_0$, there is negligible absorption of the wave.

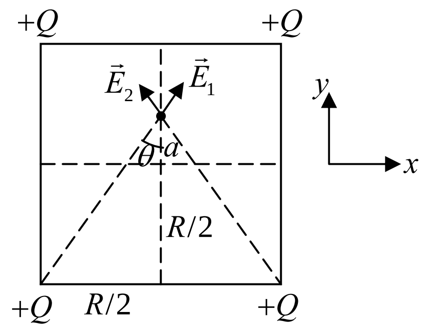
NET/JRF–(JUNE-2016)

Q70. Four equal charges of $+Q$, each are kept at the vertices of a square of side R. A particle of mass m and charge $+Q$ is placed in the plane of the square at a short distance $a(\ll R)$ from the centre. If the motion of the particle is confined to the plane, it will undergo small oscillations with an angular frequency

(a) $\sqrt{\dfrac{Q^2}{2\pi\varepsilon_0 R^3 m}}$ (b) $\sqrt{\dfrac{Q^2}{\pi\varepsilon_0 R^3 m}}$

(c) $\sqrt{\dfrac{\sqrt{2}Q^2}{\pi\varepsilon_0 R^3 m}}$ (d) $\sqrt{\dfrac{Q^2}{4\pi\varepsilon_0 R^3 m}}$

Ans. : (c)

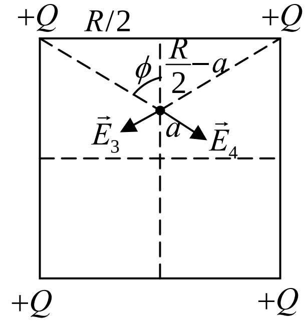


Solution: $E_1 = E_2 = \dfrac{kQ}{\left[\left(a+\dfrac{R}{2}\right)^2 + \dfrac{R^2}{4}\right]}$

Resultant field $E_{12,y} = 2E_1\cos\theta$

$$E_{12,y} = \frac{2kQ}{\left[\left(a+\frac{R}{2}\right)^2 + \frac{R^2}{4}\right]^{\frac{3}{2}}}\left(a+\frac{R}{2}\right) \approx \frac{2kQ}{\left[\frac{R^2}{2}\right]^{\frac{3}{2}}}\left(a+\frac{R}{2}\right)$$

$$E_{12,y} = \frac{4\sqrt{2}kQ}{R^3}\left(a+\frac{R}{2}\right)$$



Similarly; $E_3 = E_4 = \dfrac{kQ}{\left[\left(\dfrac{R}{2}-a\right)^2 + \dfrac{R^2}{4}\right]}$

Resultant $E_{34,y} = 2E_3\cos\phi = \dfrac{2kQ}{\left[\left(\dfrac{R}{2}-a\right)^2 + \dfrac{R^2}{4}\right]^{\frac{3}{2}}}\left(\dfrac{R}{2}-a\right)$

$$\Rightarrow \quad E_{34,y} = \frac{4\sqrt{2}kQ}{R^3}\left(\frac{R}{2}-a\right)$$

Resultant $E = \frac{4\sqrt{2}kQ}{R^3}\left[\left(\frac{R}{2}-a\right)-\left(\frac{R}{2}+a\right)\right] = -\frac{8\sqrt{2}kQ}{R^3}a$

$$E = \frac{-8\sqrt{2}}{R^3}\times\frac{1}{4\pi\varepsilon_0}Qa \Rightarrow E = -\frac{2\sqrt{2}Q}{\pi\varepsilon_0 R^3}a$$

$$\Rightarrow \quad F = QE = -\frac{2\sqrt{2}Q^2}{\pi\varepsilon_0 R^3}a \Rightarrow \omega = \sqrt{\frac{2\sqrt{2}Q^2}{\pi\varepsilon_0 m R^3}}$$

Q71. Two parallel plate capacitors, separated by distances x and $1.1x$ respectively, have a dielectric material of dielectric constant 3.0 inserted between the plates and are connected to a battery of voltage V. The difference in charge on the second capacitor compared to the first is

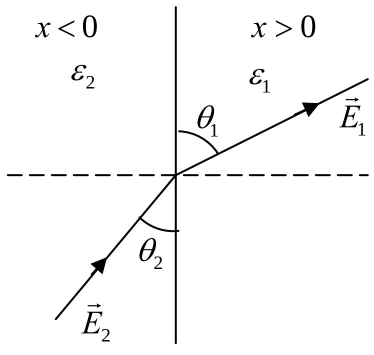
(a) +66% (b) +20% (c) −3.3% (d) −10%

Ans. : (d)

Solution: $Q_1 = C_1V_1 = \frac{3\varepsilon_0 A}{x}V$, $Q_2 = C_2V_2 = \frac{3\varepsilon_0 A}{1.1x}V$

$$\frac{Q_2 - Q_1}{Q_1}\times 100\% = \frac{\left(\frac{1}{1.1}-1\right)\times\frac{3\varepsilon_0 A}{x}V}{\frac{3\varepsilon_0 A}{x}V}\times 100 = -9\%$$

Q72. The half space region $x>0$ and $x<0$ are filled with dielectric media of dielectric constants ε_1 and ε_2 respectively. There is a uniform electric field in each part. In the right half, the electric field makes an angle θ_1 to the interface. The corresponding angle θ_2 in the left half satisfies



(a) $\varepsilon_1 \sin\theta_2 = \varepsilon_2 \sin\theta_1$ (b) $\varepsilon_1 \tan\theta_2 = \varepsilon_2 \tan\theta_1$

(c) $\varepsilon_1 \tan\theta_1 = \varepsilon_2 \tan\theta_2$ (d) $\varepsilon_1 \sin\theta_1 = \varepsilon_2 \sin\theta_2$

Ans. : (c)

Solution: $\dfrac{\tan\theta_1}{\tan\theta_2} = \dfrac{\dfrac{E_1^{\perp}}{E_1^{\parallel}}}{\dfrac{E_2^{\perp}}{E_2^{\parallel}}} = \dfrac{E_1^{\perp}}{E_2^{\perp}}$ $\quad\left(\because E_1^{\parallel} = E_2^{\parallel}\right)$

$$D_1^{\perp} = D_2^{\perp} \Rightarrow \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \Rightarrow \frac{E_1^{\perp}}{E_2^{\perp}} = \frac{\varepsilon_2}{\varepsilon_1} \Rightarrow \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_2}{\varepsilon_1} \Rightarrow \varepsilon_1 \tan\theta_1 = \varepsilon_2 \tan\theta_2$$

Q73. The x- and z-components of a static magnetic field in a region are $B_x = B_0\left(x^2 - y^2\right)$ and $B_z = 0$, respectively. Which of the following solutions for its y-component is consistent with the Maxwell equations?

(a) $B_y = B_0 xy$ (b) $B_y = -2B_0 xy$

(c) $B_y = -B_0\left(x^2 - y^2\right)$ (d) $B_y = B_0\left(\frac{1}{3}x^3 - xy^2\right)$

Ans. : (b)

Solution: $B_x = B_0\left(x^2 - y^2\right), B_z = 0$

$$\because \vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \Rightarrow \frac{\partial B_y}{\partial y} = -\frac{\partial B_x}{\partial x} = -2B_0 x \Rightarrow B_y = -2B_0 xy$$

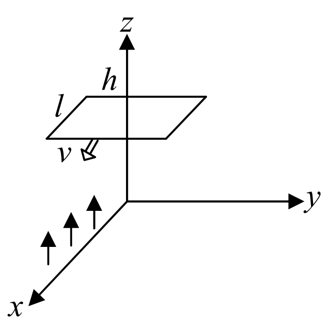
Q74. A magnetic field B is $B\hat{z}$ in the region $x > 0$ and zero elsewhere. A rectangular loop, in the xy-plane, of sides l (along the x-direction) and h (along the y- direction) is inserted into the $x > 0$ region from the $x < 0$ region at constant velocity $v = v\hat{x}$. Which of the following values of l and h will generate the largest EMF?

(a) $l = 8, h = 3$ (b) $l = 4, h = 6$ (c) $l = 6, h = 4$ (d) $l = 12, h = 2$

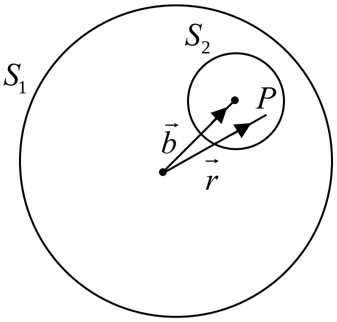
Ans. : (b)

Solution: $\phi_m \propto Bhx$

$$\varepsilon \propto \frac{-d\phi_m}{dt} \propto Bvh \propto h$$



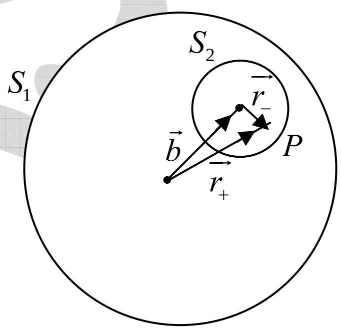
Q75. Consider a sphere S_1 of radius R which carries a uniform charge of density ρ. A smaller sphere S_2 of radius $a < \frac{R}{2}$ is cut out and removed from it. The centres of the two spheres are separated by the vector $\vec{b} = \frac{\hat{n}R}{2}$, as shown in the figure. The electric field at a point P inside S_2 is



(a) $\frac{\rho R}{3\varepsilon_0}\hat{n}$ (b) $\frac{\rho R}{3\varepsilon_0 a}(\vec{r} - \hat{n}a)$ (c) $\frac{\rho R}{6\varepsilon_0}\hat{n}$ (d) $\frac{\rho a}{3\varepsilon_0 R}\vec{r}$

Ans. : (c)

Solution: Electric field at P due to S_1 is $\vec{E}_1 = \frac{\rho}{3\varepsilon_0}\vec{r}_+$



Electric field at P due to S_2 (assume $-\rho$) is $\vec{E}_2 = \frac{-\rho}{3\varepsilon_0}\vec{r}_-$

Thus $\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}(\vec{r}_+ - \vec{r}_-)$; $\because \vec{b} + \vec{r}_- = \vec{r}_+ \Rightarrow \vec{r}_+ - \vec{r}_- = \vec{b}$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{b} = \frac{\rho R}{6\varepsilon_0}\hat{n} \quad \left(\because \vec{b} = \frac{R}{2}\hat{n}\right)$$

Q76. The value of the electric and magnetic fields in a particular reference frame (in Gaussian units) are $E = 3\hat{x} + 4\hat{y}$ and $B = 3\hat{z}$ respectively. An inertial observer moving with respect to this frame measures the magnitude of the electric field to be $|E'| = 4$. The magnitude of the magnetic field $|B'|$ measured by him is

(a) 5 (b) 9 (c) 0 (d) 1

Ans. : (c)

Solution: $\because E^2 - B^2 = E'^2 - B'^2 = \text{constant} \Rightarrow (9+16) - 9 = 16 - B'^2 \Rightarrow B' = 0$

Q77. A loop of radius a, carrying a current I, is placed in a uniform magnetic field B. If the normal to the loop is denoted by $\hat{n}$, the force $\vec{F}$ and the torque $\vec{T}$ on the loop are

(a) $\vec{F} = 0$ and $\vec{T} = \pi a^2 I\,\hat{n} \times B$ (b) $\vec{F} = \frac{\mu_0}{4\pi}\vec{I} \times \vec{B}$

(c) $\vec{F} = \frac{\mu_0}{4\pi}\vec{I} \times \vec{B}$ and $\vec{T} = I\,\hat{n} \times \vec{B}$ (d) $\vec{F} = 0$ and $\vec{T} = \frac{1}{\mu_0\varepsilon_0}I\vec{B}$

Ans. : (a)

Solution: In uniform field $\vec{F}=0$

Torque $\vec{T}=\vec{m}\times\vec{B}=\pi a^2 I\hat{n}\times\vec{B}$

Q78. A waveguide has a square cross-section of side $2a$. For the TM modes of wave vector k, the transverse electromagnetic modes are obtained in terms of a function $\psi(x,y)$ which obeys the equation

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\left(\frac{\omega^2}{c^2}-k^2\right)\right]\psi(x,y)=0$$

with the boundary condition $\psi(\pm a,y)=\psi(x,\pm a)=0$. The frequency ω of the lowest mode is given by

(a) $\omega^2=c^2\left(k^2+\frac{4\pi^2}{a^2}\right)$ (b) $\omega^2=c^2\left(k^2+\frac{\pi^2}{a^2}\right)$

(c) $\omega^2=c^2\left(k^2+\frac{\pi^2}{2a^2}\right)$ (d) $\omega^2=c^2\left(k^2+\frac{\pi^2}{4a^2}\right)$

Ans. : (c)

Solution: $c^2k^2=\omega^2-\omega_{mn}^2 \Rightarrow \omega^2=c^2k^2+\omega_{mn}^2$

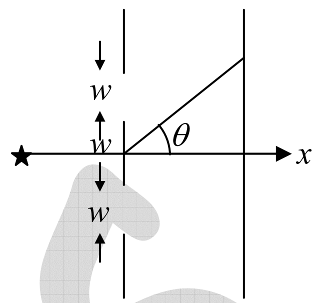
$$\Rightarrow \omega_{mn}^2=c^2\pi^2\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\Rightarrow \omega_{11}^2=c^2\pi^2\left[\frac{1}{(2a)^2}+\frac{1}{(2a)^2}\right]$$

$$\Rightarrow \omega_{11}^2=c^2\pi^2\times\frac{1}{2a^2}=\frac{c^2\pi^2}{2a^2} \Rightarrow \omega^2=c^2\left(k^2+\frac{\pi^2}{2a^2}\right)$$

NET/JRF -(DEC-2016)

Q79. A screen has two slits, each of width w with their centres at a distance $2w$ apart. It is illuminated by a monochromatic plane wave travelling along the x-axis.

The intensity of the interference pattern, measured on a distant screen, at an angle $\theta = \frac{n\lambda}{w}$ to the x-axis is



(a) zero for $n = 1, 2, 3...$

(b) maximum for $n = 1, 2, 3...$

(c) maximum for $n = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}...$

(d) zero for $n = 0$ only

Ans. : (a)

Solution: maximum for $n = 0$ and zero for $n = 1, 2, 3....$

Q80. The electric field of an electromagnetic wave is

$$\vec{E}(z,t) = E_0 \cos(kz + \omega t)\hat{i} + 2E_0 \sin(kz + \omega t)\hat{j}$$

where ω and k are positive constants. This represents

(a) a linearly polarised wave travelling in the positive z-direction

(b) a circularly polarised wave travelling in the negative z-direction

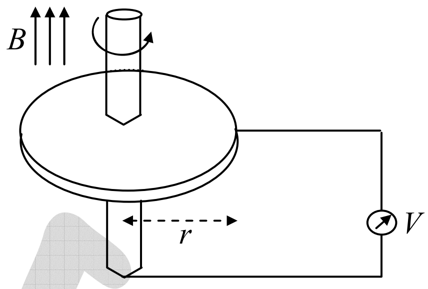
(c) an elliptically polarised wave travelling in the negative z-direction

(d) an unpolarised wave travelling in the positive z-direction

Ans. : (c)

Solution: Amplitude along $\hat{i}$ is E_0 and along $\hat{j}$ is $2E_0$. So resultant wave is elliptically polarised

Q81. A conducting circular disc of radius r and resistivity ρ rotates with an angular velocity ω in a magnetic field B perpendicular to it. A voltmeter is connected as shown in the figure below. Assuming its internal resistance to be infinite, the reading on the voltmeter



(a) depends on ω, B, r and ρ

(b) depends on ω, B and r but not on ρ

(c) is zero because the flux through the loop is not changing

(d) is zero because a current the flows in the direction of B

Ans. : (b)

Solution: Force experienced by charge is

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \text{ and } v = r\omega$$

Q82. The charge per unit length of a circular wire of radius a in the xy-plane, with its centre at the origin, is $\lambda = \lambda_0 \cos\theta$, where λ_0 is a constant and the angle θ is measured from the positive x-axis. The electric field at the centre of the circle is

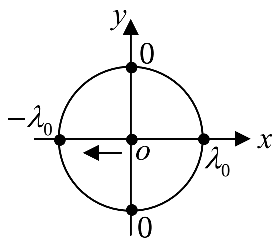
(a) $\vec{E} = -\dfrac{\lambda_0}{4\epsilon_0 \alpha}\hat{i}$ (b) $\vec{E} = \dfrac{\lambda_0}{4\epsilon_0 \alpha}\hat{i}$

(c) $\vec{E} = -\dfrac{\lambda_0}{4\epsilon_0 \alpha}\hat{j}$ (d) $\vec{E} = \dfrac{\lambda_0}{4\pi\epsilon_0 \alpha}\hat{k}$

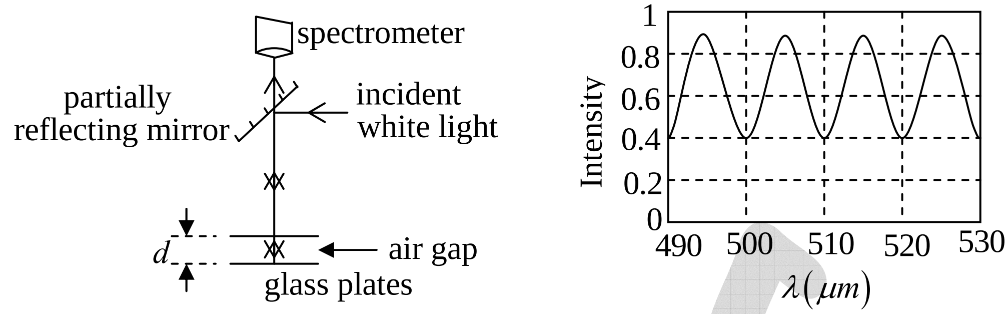
Ans. : (a)

Solution: At centre O, direction of field is $-\hat{x}$.

So best option is (a)



Q83. A pair of parallel glass plates separated by a distance d is illuminated by white light as shown in the figure below. Also shown in the graph of the intensity of the reflected light I as a function of the wavelength λ recorded by a spectrometer.



Assuming that the interference takes place only between light reflected by the bottom surface of the top plate and the top surface of bottom plate, the distance d is closest to

(a) 12 μm (b) 24 μm (c) 60 μm (d) 120 μm

Ans. : (d)

Solution: For constructive interference of reflected light, $2d\cos\theta = \left(n+\frac{1}{2}\right)\lambda$.

First maxima occurs at $\lambda = 495\ \mu m$, $\theta = 0^0$ and $n = 0$. Thus, $d = \frac{\lambda}{4} = \frac{495\ \mu m}{4} \approx 120\ \mu m$

Q84. Suppose that free charges are present in a material of dielectric constant $\in = 10$ and resistivity $\rho = 10^{11}\ \Omega - m$. Using Ohm's law and the equation of continuity for charge, the time required for the charge density inside the material to decay by $\frac{1}{e}$ is closest to

(a) $10^{-6}\ S$ (b) $10^{6}\ S$ (c) $10^{12}\ S$ (d) $10\ S$

Ans. : (d)

Solution: $\rho_f(t) = \rho_f(0)e^{\frac{-t}{\tau}}$; $\tau = \frac{\in}{\sigma} = \frac{\in_0 \in_r}{\sigma}$, $\tau = \frac{8.8\times 10^{-12}\times 10}{10^{-11}} \approx 10\sec$, $\sigma = \frac{1}{\rho}$

Q85. A particle with charge $-q$ moves with a uniform angular velocity ω in a circular orbit of radius a in the xy- plane, around a fixed charge $+q$, which is at the centre of the orbit at $(0,0,0)$. Let the intensity of radiation at the point $(0,0,R)$ be I_1 and at $(2R,0,0)$ be 'I_2

The ratio $\frac{I_2}{I_1}$ for $R \gg a$, is

(a) 4 (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) 8

Ans. : (c)

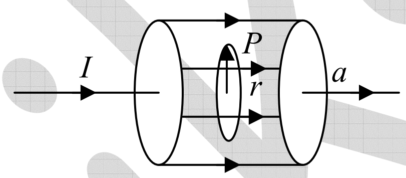
Solution: $\frac{I_2}{I_1}=\frac{r_1^3}{r_2^3}=\frac{R^3}{(2R)^3}=\frac{1}{8}$

Q86. A parallel plate capacitor is formed by two circular conducting plates of radius a separated by a distance d, where $d \ll a$. It is being slowly charged by a current that is nearly constant. At an instant when the current is I, the magnetic induction between the plates at a distance $\frac{a}{2}$ from the centre of the plate, is

(a) $\frac{\mu_0 I}{\pi a}$ (b) $\frac{\mu_0 I}{2\pi a}$ (c) $\frac{\mu_0 I}{a}$ (d) $\frac{\mu_0 I}{4\pi a}$

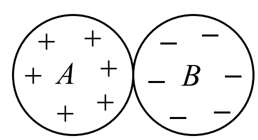
Ans. : (d)

Solution: $\left|\vec{B}\right|=\frac{\mu_0 I r}{2\pi a^2}$



$\left|\vec{B}\right|=\frac{\mu_0 I}{4\pi a}$ at $r=\frac{a}{2}$

Q87. Two uniformly charged insulating solid spheres A and B, both of radius a, carry total charges $+Q$ and $-Q$, respectively. The spheres are placed touching each other as shown in the figure.



If the potential at the centre of the sphere A is V_A and that at the centre of B is V_B then the difference $V_A - V_B$ is

(a) $\frac{Q}{4\pi\varepsilon_0 a}$ (b) $\frac{-Q}{2\pi\varepsilon_0 a}$ (c) $\frac{Q}{2\pi\varepsilon_0 a}$ (d) $\frac{-Q}{4\pi\varepsilon_0 a}$

Ans. : (c)

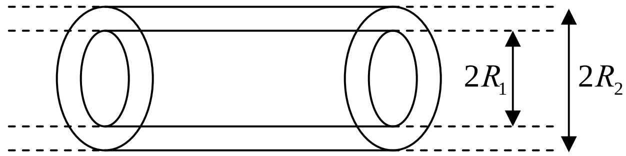
Solution: $V_A=\frac{3Q}{8\pi \epsilon_0 a}-\frac{Q}{4\pi \epsilon_0 (2a)}=\frac{Q}{4\pi \epsilon_0 a}$

$V_B=\frac{-3Q}{8\pi \epsilon_0 a}+\frac{Q}{4\pi \epsilon_0 (2a)}=\frac{-Q}{4\pi \epsilon_0 a}$

$V_A - V_B=\frac{Q}{2\pi \epsilon_0 a}$

NET/JRF -(JUNE -2017)

Q88. Two long hollow co-axial conducting cylinders of radii R_1 and R_2 $(R_1 < R_2)$ are placed in vacuum as shown in the figure below.



The inner cylinder carries a charge $+\lambda$ per unit length and the outer cylinder carries a charge $-\lambda$ per unit length. The electrostatic energy per unit length of this system is

(a) $\frac{\lambda^2}{\pi \in_0} \ln(R_2/R_1)$ (b) $\frac{\lambda^2}{4\pi \in_0}(R_2^2/R_1^2)$

(c) $\frac{\lambda^2}{4\pi \in_0} \ln(R_2/R_1)$ (d) $\frac{\lambda^2}{2\pi \in_0} \ln(R_2/R_1)$

Ans. : (c)

Solution: $r < R_1, \vec{E}_1 = 0$; $R_1 < r < R_2, \vec{E}_2 = \frac{\lambda}{2\pi \in_0 r}\hat{r}$

$r > R_z, \ \vec{E}_3 = 0$

$$W = \frac{\in_0}{2}\int_{\text{all spce}} E^2 dz = \frac{\in_0}{2}\int_{R_1}^{Rz} \frac{\lambda^2}{4\pi^2 \in_0^2 r^2} \times 2\pi r l dr$$

$$\frac{W}{l} = \frac{\in_0}{2} \times \frac{\lambda^2}{2\pi \in_0^2}\int_{R_1}^{R_2} \frac{1}{r} dr = \frac{\lambda^2}{4\pi \in_0} \ln\left(\frac{R_2}{R_1}\right)$$

Q89. A set of N concentric circular loops of wire, each carrying a steady current I in the same direction, is arranged in a plane. The radius of the first loop is $r_1 = a$ and the radius of the n^{th} loop is given by $r_n = n r_{n-1}$. The magnitude B of the magnetic field at the centre of the circles in the limit $N \to \infty$, is

(a) $\mu_0 I(e^2 - 1)/4\pi a$ (b) $\mu_0 I(e-1)/\pi a$

(c) $\mu_0 I(e^2 - 1)/8a$ (d) $\mu_0 I(e-1)/2a$

Ans. : (d)

Solution: $B = \frac{\mu_0 I}{2}\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ...\frac{1}{r_n}\right)$

$r_1 = a$

$r_n = nr_{n-1}$

$r_1 = r_0 = a$, $r_2 = 2r_1 = 2a$, $r_3 = 3r_2 = 3.2a$ and $r_4 = 4r_3 = 4.3.2a$

$\Rightarrow B = \frac{\mu_0 I}{2a}\left(1 + \frac{1}{2} + \frac{1}{3.2} + \frac{1}{4.3.2} + ...\right)$

$B = \frac{\mu_0 I}{2a}\left(\sum_{n=1}^{N} \frac{1}{\lfloor n}\right)$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{\lfloor n} \Rightarrow e = \sum_{n=0}^{\infty} \frac{1}{\lfloor n} = 1 + \sum_{n=1}^{\infty} \frac{1}{\lfloor n} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{\lfloor n} = e - 1$

$\lim_{N \to \infty}\left(\sum_{n=l}^{N} \frac{1}{\lfloor n}\right) = e - 1 \Rightarrow B = \frac{\mu_0 I}{2a}(e-1)$

Q90. An electromagnetic wave (of wavelength λ_0 in free space) travels through an absorbing medium with dielectric permittivity given by $\varepsilon = \varepsilon_R + i\varepsilon_I$ where $\frac{\varepsilon_I}{\varepsilon_R} = \sqrt{3}$. If the skin depth is $\frac{\lambda_0}{4\pi}$, the ratio of the amplitude of electric field E to that of the magnetic field B, in the medium (in ohms) is

(a) $120\,\pi$ (b) 377 (c) $30\sqrt{2}\,\pi$ (d) 30π

Ans. : (d)

Solution: $d = \frac{1}{\chi} = \frac{\lambda_0}{4\pi}$, $\frac{\in_I}{\in_R} = \sqrt{3} = \frac{\sigma}{\omega \in}$

$\chi = \omega\sqrt{\frac{\in \mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega \in}\right)^2} - 1\right]^{1/2} \Rightarrow \chi = \omega\sqrt{\frac{\in \mu}{2}} = \frac{4\pi}{\lambda_0} \Rightarrow \sqrt{\in \mu} = \frac{\sqrt{2}}{\omega}\frac{4\pi}{\lambda_0}$

$K = \sqrt{k^2 + \chi^2} = \omega\left[\in \mu\sqrt{1 + \left(\frac{\sigma}{\omega \in}\right)^2}\right]^{1/2}$

$$\frac{E_0}{B_0}=\frac{\omega}{K}=\frac{\omega}{\omega\left[\in\mu\sqrt{1+\left(\frac{\sigma}{\omega\in}\right)^2}\right]^{1/2}}=\frac{1}{\sqrt{2\in\mu}}=\frac{1}{\sqrt{2}\times\frac{\sqrt{2}}{\omega}\times\frac{4\pi}{\lambda_0}}$$

$$=\frac{\lambda_0\omega}{8\pi}=\frac{\lambda_0\times 2\pi c/\lambda_0}{8\pi}=\frac{c}{4}\Rightarrow\frac{E}{H_0}=\frac{c}{4}\mu_0$$

$$\Rightarrow\frac{E}{H_0}=\frac{4\pi\times10^{-7}\times3\times10^{8}}{4}=30\pi$$

Q91. The vector potential $\vec{A}=ke^{-at}r\hat{r}$ (where a and k are constants) corresponding to an electromagnetic field is changed to $\vec{A}'=-ke^{-at}r\hat{r}$. This will be a gauge transformation if the corresponding change $\phi'-\phi$ in the scalar potential is

(a) akr^2e^{-at} (b) $2akr^2e^{-at}$ (c) $-akr^2e^{-at}$ (d) $-2akr^2e^{-at}$

Ans. : (c)

Solution: Gauge Transformation

$$\vec{A}=\vec{A}+\vec{\nabla}\lambda,\ \phi'=\phi-\frac{\partial\lambda}{\partial t}\Rightarrow\vec{A}'-\vec{A}=-2ke^{-at}r\hat{r}=\vec{\nabla}\lambda=\frac{\partial\lambda}{\partial r}\hat{r}$$

$$\Rightarrow\lambda=-ke^{-at}r^2\Rightarrow\frac{\partial\lambda}{\partial t}=kae^{-at}r^2$$

$$\Rightarrow\phi'-\phi=-\frac{\partial\lambda}{\partial t}=-kae^{-at}r^2$$

Q92. An electron is decelerated at a constant rate starting from an initial velocity u (where $u<<c$) to $u/2$ during which it travels a distance s. The amount of energy lost to radiation is

(a) $\frac{\mu_0e^2u^2}{3\pi mc^2s}$ (b) $\frac{\mu_0e^2u^2}{6\pi mc^2s}$ (c) $\frac{\mu_0e^2u}{8\pi mcs}$ (d) $\frac{\mu_0e^2u}{16\pi mcs}$

Ans. : (d)

Solution: Total power radiated $P=\frac{\mu_0q^2a^2}{6\pi c}$

Total energy radiated in time t is $E=P\cdot t=\frac{\mu_0e^2a^2}{6\pi c}\cdot t=\frac{\mu_0e^2a^2}{6\pi c}\times\frac{u}{2a}$

$$\left[\because v = u - at \Rightarrow \frac{u}{2} = u - at \Rightarrow t = \frac{u}{2a}\right]$$

$$\Rightarrow E = \frac{\mu_0 e^2 au}{12\pi c}$$

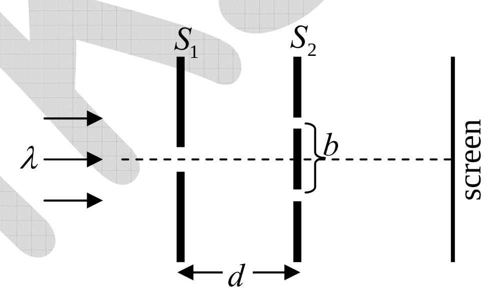
Fraction of initial $K.E.$ lost due to radiation $= \dfrac{E}{\frac{1}{2}mu^2} = \dfrac{2E}{mu^2}$

$$= \frac{2}{mu^2} \times \frac{\mu_0 e^2 au}{12\pi c} = \frac{\mu_0 e^2 a}{6\pi mcu}$$

$$\left[\therefore s = ut - \frac{1}{2}at^2 = u \times \frac{u}{2a} - \frac{1}{2}a \times \frac{u^2}{4a^2} = \frac{u^2}{2a} - \frac{u^2}{8a} = \frac{3u^2}{8a} \Rightarrow a = \frac{3u^2}{8s}\right]$$

$$= \frac{\mu_0 e^2}{6\pi mcu} \times \frac{3u^2}{8s} = \frac{\mu_0 e^2 u}{16\pi mcs}$$

Q93. The figure describes the arrangement of slits and screens in a Young's double slit experiment. The width of the slit in S_1 is a and the slits in S_2 are of negligible width.



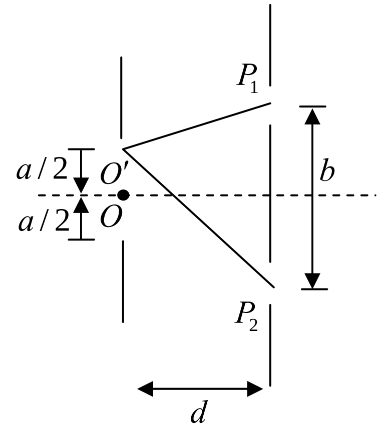
If the wavelength of the light is λ, the value of d for which the screen would be dark is

(a) $b\sqrt{\left(\frac{a}{\lambda}\right)^2 - 1}$ (b) $\frac{b}{2}\sqrt{\left(\frac{a}{\lambda}\right)^2 - 1}$ (c) $\frac{a}{2}\left(\frac{b}{\lambda}\right)^2$ (d) $\frac{ab}{\lambda}$

Ans. : (d)

Solution: If the path difference $O'p_2 - O'p_1 = \dfrac{\lambda}{2}$

The minima of the interference pattern produced by O will fall on the maxima produced by O' Now



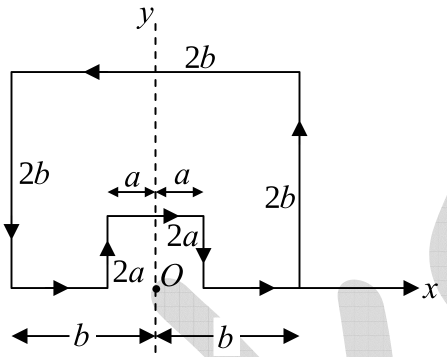
$$O'P_2 = \left[d^2 + \left(\frac{b}{2} + \frac{a}{2}\right)^2\right]^{1/2} \approx d + \frac{1}{2d}\left(\frac{b}{2} + \frac{a}{2}\right)^2$$

$$O'P_1 = \left[d^2 + \left(\frac{b}{2} - \frac{a}{2}\right)^2\right]^{1/2} \approx d + \frac{1}{2d}\left(\frac{b}{2} - \frac{a}{2}\right)^2$$

$$\Rightarrow O'P_2 - O'P_1 \approx \frac{ab}{2d} \quad (\because d >> b, a)$$

Thus $\frac{\lambda}{2} = \frac{ab}{2d} \Rightarrow d = \frac{ab}{\lambda}$

Q94. A constant current I is flowing in a piece of wire that is bent into a loop as shown in the figure.



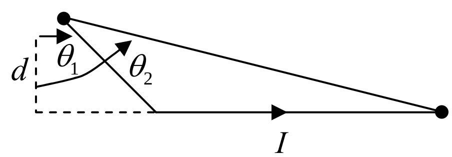
The magnitude of the magnetic field at the point O is

(a) $\frac{\mu_0 I}{4\pi\sqrt{5}} \ln\left(\frac{a}{b}\right)$ (b) $\frac{\mu_0 I}{4\pi\sqrt{5}}\left(\frac{1}{a} - \frac{1}{b}\right)$ (c) $\frac{\mu_0 I}{4\pi\sqrt{5}}\left(\frac{1}{a}\right)$ (d) $\frac{\mu_0 I}{4\pi\sqrt{5}}\left(\frac{1}{b}\right)$

Ans. : (b)

Solution: $\vec{B} = \frac{\mu_0 I}{4\pi d}(\sin\theta_2 - \sin\theta_1)\hat{\phi}$

Magnetic field due to left and right segment of $2a$



$$B_{2a} = \frac{\mu_0 I}{4\pi a}\left(\frac{2a}{\sqrt{5}a}\right) \otimes$$

Field due to upper segment of $2a$

$$= \frac{\mu_0 I}{4\pi(2a)} \times \left(\frac{a}{\sqrt{5}a} + \frac{a}{\sqrt{5}a}\right)$$

Net field $B_{2a} = 2 \times \frac{\mu_0 I}{4\pi a} \times \frac{2}{\sqrt{5}} + \frac{\mu_0 I}{4\pi a} \times \frac{1}{\sqrt{5}}$

$$B_{2a} = \frac{\mu_0 I}{4\pi a}\sqrt{5} \otimes \text{ (inward)}$$

similarly, $B_{2b} = \frac{\mu_0 I}{4\pi b}\sqrt{5} \odot$ (outward)

Net field $B = B_{2a} - B_{2b} = \frac{\mu_0 I}{4\pi}\sqrt{5}\left(\frac{1}{a} - \frac{1}{b}\right)$

Q95. The charge distribution inside a material of conductivity σ and permittivity $\in$ at initial time $t=0$ is $\rho(r,0)=\rho_0$, a constant. At subsequent times $\rho(r,t)$ is given by

(a) $\rho_0 \exp\left(-\frac{\sigma t}{\in}\right)$

(b) $\frac{1}{2}\rho_0\left[1+\exp\left(\frac{\sigma t}{\in}\right)\right]$

(c) $\frac{\rho_0}{\left[1-\exp\left(\frac{\sigma t}{\in}\right)\right]}$

(d) $\rho_0 \cosh\frac{\sigma t}{\in}$

Ans. : (a)

Solution: $\vec{J}_f=\sigma\vec{E}, \vec{\nabla}.\vec{E}=\frac{\rho_f}{\in}, \quad \vec{\nabla}.\vec{J}_f=-\frac{\partial\rho_f}{\partial E}$

$$\Rightarrow \sigma\vec{\nabla}.\vec{E}=-\frac{\partial\rho_f}{\partial t}\Rightarrow\frac{\partial\rho_f}{\partial t}=-\frac{\sigma}{\in}\rho_f$$

$$\Rightarrow \rho_f(t)=\rho_0\exp\left(\frac{-\sigma}{\in}\rho_f\right)\Rightarrow\rho_f(t)=\rho_0\exp\left(\frac{-\sigma}{\in}t\right)$$

NET/JRF (DEC - 2017)

Q96. Two point charges $+3Q$ and $-Q$ are placed at $(0,0,d)$ and $(0,0,2d)$ respectively, above an infinite grounded conducting sheet kept in the xy- plane. At a point $(0,0,z)$, where $z>>d$, the electrostatic potential of this charge configuration would approximately be

(a) $\frac{1}{4\pi\varepsilon_0}\frac{d^2}{z^3}Q$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{2d}{z^2}Q$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{3d}{z^2}Q$ (d) $-\frac{1}{4\pi\varepsilon_0}\frac{d^2}{z^3}Q$

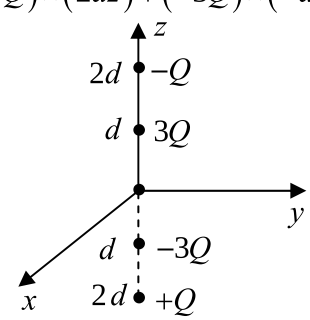
Ans. : (b)

Solution: Monopole moment $Q_{mono}=-Q+3Q-3Q+Q=0$

Dipole moment $\vec{p}=+3Q\times(d\hat{z})+(-Q)\times(2d\hat{z})+(-3Q)\times(-d\hat{z})+Q\times(-2d\hat{z})$

$\vec{p}=2Qd\hat{z}$

$$V_{dip}=\frac{1}{4\pi\in_0}\frac{\vec{p}\cdot\hat{r}}{r^2}\approx\frac{1}{4\pi\in_0}\frac{2Qd}{z^2}$$



Q97. A rectangular piece of dielectric material is inserted partially into the (air) gap between the plates of a parallel plate capacitor. The dielectric piece will

(a) remain stationary where it is placed

(b) be pushed out from the gap between the plates

(c) be drawn inside the gap between the plates and its velocity does not change sign

(d) execute an oscillatory motion in the region between the plates

Ans. : (c)

Solution: Just like a conductor, a dielectric is attracted into an electric field. The reason is: the bound charge tends to accumulate near the fee charge of the opposite sign.

Q98. An electromagnetic wave is travelling in free space (of permittivity ε_0) with electric field

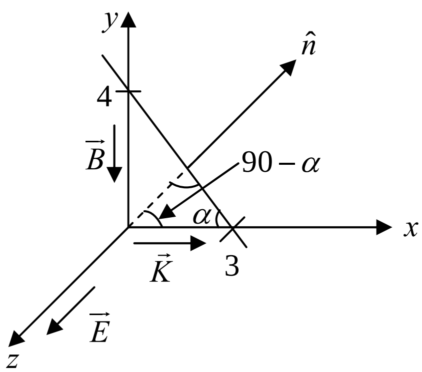
$$\vec{E} = \hat{k}E_0 \cos q\left(x - ct\right)$$

The average power (per unit area) crossing planes parallel to $4x + 3y = 0$ will be

(a) $\frac{4}{5}\varepsilon_0 c E_0^2$ (b) $\varepsilon_0 c E_0^2$ (c) $\frac{1}{2}\varepsilon_0 c E_0^2$ (d) $\frac{16}{25}\varepsilon_0 c E_0^2$

Ans. : (c)

Solution: $4x + 3y = 0 \Rightarrow \frac{x}{3} + \frac{y}{4} = 0$



$$\vec{B} = -\frac{E_0}{c}\cos\left(qx - qct\right)\hat{y}$$

$$\vec{S} = \frac{1}{\mu_0}\left(\vec{E}\times\vec{B}\right) = \frac{1}{\mu_0}\left(E_0 \times \frac{E_0}{c}\cos^2\theta\right)\hat{x} \Rightarrow \left\langle \vec{S}\right\rangle = \frac{E_0^2}{2\mu_0 c}\hat{x}$$

$$I = \left\langle \vec{S}\right\rangle . \hat{n} = \frac{E_0^2}{2\mu_0 c}\cos\left(90 - \alpha\right) = \frac{E_0^2}{2\mu_0 c}\sin\alpha = \frac{2}{5}c\varepsilon_0 E_0^2$$

$$\because \tan\alpha = \frac{4}{3} \Rightarrow \sin\alpha = \frac{4}{5}$$

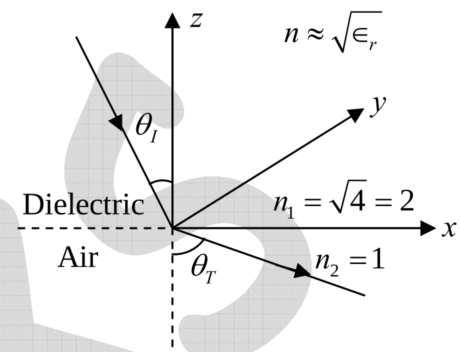
$$I \approx 0.4 c\varepsilon_0 E_0^2 \approx \frac{1}{2}c\varepsilon_0 E_0^2$$

Q99. A plane electromagnetic wave from within a dielectric medium (with $\varepsilon = 4\varepsilon_0$ and $\mu = \mu_0$) is incident on its boundary with air, at $z = 0$. The magnetic field in the medium is $\vec{H} = \hat{j}H_0 \cos\left(\omega t - kx - k\sqrt{3}\,z\right)$, where ω and k are positive constants.

The angles of reflection and refraction are, respectively,

(a) 45^0 and 60^0 (b) 30^0 and 90^0 (c) 30^0 and 60^0 (d) 60^0 and 90^0

Ans. : (b)



Solution: $n \approx \sqrt{\in_r}$

$$\vec{k} = k\hat{x} + k\sqrt{3}\hat{z}$$

$$\frac{\sin\theta_I}{\sin\theta_T} = \frac{n_2}{n_1} = \frac{1}{2}$$

$$\sin\theta_T = 2\sin\theta_I \qquad \because \tan\theta_I = \frac{k_x}{k_z} = \frac{1}{\sqrt{3}} \Rightarrow \theta_I = 30^0$$

$$\Rightarrow \sin\theta_T = 2\times\sin 30^0 = 1 \Rightarrow \theta_T = 90^0$$

Q100. In an inertial frame S, the magnetic vector potential in a region of space is given by $\vec{A} = az\,\hat{i}$ (where a is a constant) and the scalar potential is zero. The electric and magnetic fields seen by an inertial observer moving with a velocity $v\hat{i}$ with respect to S, are, respectively [In the following $\gamma = \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}}$]

(a) 0 and $\gamma a\hat{j}$ (b) $-va\hat{k}$ and $\gamma a\hat{i}$ (c) $v\gamma a\hat{k}$ and $v\gamma a\hat{j}$ (d) $v\gamma a\hat{k}$ and $\gamma a\hat{j}$

Ans. : (d)

Solution: $E_x = E'_x,\ E_y = \gamma\left(E'_y + vB'_z\right)$ and $E_z = \gamma\left(E'_z - vB'_y\right)$

$$B_x = B'_x, B_y = \gamma\left(B'_y - \frac{v}{c^2}E'_z\right) \text{ and } B_z = \gamma\left(B'_z + \frac{v}{c^2}E'_y\right)$$

$$\vec{E}' = -\vec{\nabla}V' - \frac{\partial\vec{A}'}{\partial t} = 0,\ \vec{B}' = \vec{\nabla}\times\vec{A}' = a\,\hat{j}$$

$$E_x = 0, E_y = \gamma\left(0 - v\times 0\right) = 0,\ E_z = \gamma\left(0 + va\right) = \gamma va$$

(replace v by $-v$) $\Rightarrow\ \vec{E} = v\gamma a\hat{z}$

$B_x = 0,\ B_y = \gamma\left(a + \frac{v}{c^2}\times 0\right) = \gamma a,\ B_z = \gamma\left(0 - \frac{v}{c^2}\times 0\right) = 0$

$\Rightarrow \vec{B} = \gamma a\,\hat{j}$

Q101. In the rest frame S_1 of a point particle with electric charge q_1 another point particle with electric charge q_2 moves with a speed v parallel to the x-axis at a perpendicular distance l. The magnitude of the electromagnetic force felt by q_1 due to q_2 when the distance between them is minimum, is [In the following $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$]

(a) $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\gamma l^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{\gamma q_1q_2}{l^2}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{\gamma q_1q_2}{l^2}\left(1+\frac{v^2}{c^2}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{\gamma l^2}\left(1+\frac{v^2}{c^2}\right)$

Ans. : (b)

Solution: Charge of q_2 seen by rest frame of $q_1 = \gamma q_2$; $F = \frac{1}{4\pi\varepsilon_0}\frac{\gamma q_1q_2}{l^2}$

Q102. A circular current carrying loop of radius a carries a steady current. A constant electric charge is kept at the centre of the loop. The electric and magnetic fields, $\vec{E}$ and $\vec{B}$ respectively, at a distance d vertically above the centre of the loop satisfy

(a) $\vec{E}\perp\vec{B}$ (b) $\vec{E} = 0$ (c) $\vec{\nabla}\left(\vec{E}\cdot\vec{B}\right) = 0$ (d) $\vec{\nabla}\cdot\left(\vec{E}\times\vec{B}\right) = 0$

Ans. : (c)

Solution: $\vec{E}\times\vec{B} = 0 \Rightarrow \vec{\nabla}.\left(\vec{E}\times\vec{B}\right) = 0$

Q103. The Hamiltonian of a two-level quantum system is $H = \frac{1}{2}\hbar\omega\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$ possible initial state in which the probability of the system being in that quantum state does not change with time, is

(a) $\begin{pmatrix} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \end{pmatrix}$ (b) $\begin{pmatrix} \cos\frac{\pi}{8} \\ \sin\frac{\pi}{8} \end{pmatrix}$ (c) $\begin{pmatrix} \cos\frac{\pi}{2} \\ \sin\frac{\pi}{2} \end{pmatrix}$ (d) $\begin{pmatrix} \cos\frac{\pi}{6} \\ \sin\frac{\pi}{6} \end{pmatrix}$

Ans. : (b)

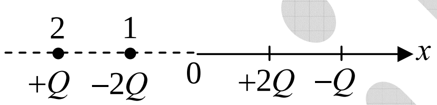
NET/JRF -(JUNE -2018)

Q104. Two point charges $+2Q$ and $-Q$ are kept at point with Cartesian coordinates $(1,0,0)$, respectively, in front of an infinite grounded conducting plate at $x=0$. The potential at $(x,0,0)$ for $x \gg 1$ depends on x as

(a) x^{-3} (b) x^{-5} (c) x^{-2} (d) x^{-4}

Ans. : (a)

Solution:

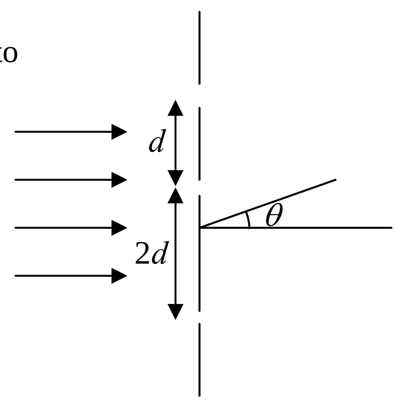


Monopole moment $2Q-Q-2Q+Q=0$

Dipole moment $\vec{p}=-Q(2\hat{x})+2Q(\hat{x})-2Q(-\hat{x})+Q(-2\hat{x}) \Rightarrow \vec{p}=0$

Thus $V \propto \frac{1}{x^3}$

Q105. The following configuration of three identical narrow slits are illuminated by monochromatic light of wavelength λ (as shown in the figure below). The intensity is measured at an angle θ (where θ is the angle with the incident beam) at a large distance from the slits. If $\delta=\frac{2\pi d}{\lambda}\sin\theta$, the intensity is proportional to



(a) $2\cos\delta+2\cos 2\delta$ (b) $3+\frac{1}{\delta^2}\sin^2 3\delta$

(c) $3+2\cos\delta+2\cos 2\delta+2\cos 3\delta$ (d) $2+\frac{1}{\delta^2}\sin^2 3\delta$

Ans. : (c)

Solution: $\vec{E}_1=\vec{A}e^{i(\omega t)}$, $\vec{E}_2=\vec{A}\,e^{i\delta}\,e^{i\omega t}$, $\vec{E}_3=\vec{A}\,e^{i\delta_1}\,e^{i\omega t}=\vec{A}\,e^{3i\delta}\,e^{i\omega t}$

$\because \delta=\frac{2\pi}{\lambda}d\sin\theta$, $\delta_1=\frac{2\pi}{\lambda}(3d\sin\theta)\approx 3\delta$

$$\vec{E}=\vec{E}_1+\vec{E}_2+\vec{E}_3=\vec{A}\left[1+e^{i\delta}+e^{3i\delta}\right]e^{i\omega t}$$

$$\vec{E}^*=\vec{A}'\left[1+e^{-i\delta}+e^{-3i\delta}\right]e^{-i\omega t}$$

$$I=\vec{E}.\vec{E}^*=A^2\left[1+e^{i\delta}+e^{3i\delta}\right]\left[1+e^{-i\delta}+e^{-3i\delta}\right]$$

$$I=A^2\left[3+2\frac{e^{i\delta}+e^{-i\delta}}{2}+2\frac{e^{i2\delta}+e^{-i2\delta}}{2}+2\frac{e^{i3\delta}+e^{-i3\delta}}{2}\right]$$

$$I=A^2\left[3+2\cos\delta+2\cos 2\delta+2\cos 3\delta\right]$$

Q106. The electric field $\vec{E}$ and the magnetic field $\vec{B}$ corresponding to the scalar and vector potentials, $V(x,y,z,t)=0$ and $\vec{A}(x,y,z,t)=\frac{1}{2}\hat{k}\mu_0 A_0(ct-x)$, where A_0 is a constant, are

(a) $\vec{E}=0$ and $\vec{B}=\frac{1}{2}\hat{j}\mu_0 A_0$

(b) $\vec{E}=-\frac{1}{2}\hat{k}\mu_0 A_0 c$ and $\vec{B}=\frac{1}{2}\hat{j}\mu_0 A_0$

(c) $\vec{E}=0$ and $\vec{B}=-\frac{1}{2}\hat{i}\mu_0 A_0$

(d) $\vec{E}=\frac{1}{2}\hat{k}\mu_0 A_0 c$ and $\vec{B}=-\frac{1}{2}\hat{i}\mu_0 A_0$

Ans. : (b)

Solution: $\vec{E}=\frac{-\partial\vec{A}}{\partial t}=-\left[\frac{1}{2}\mu_0 A_0(c-0)\right]\hat{k}=-\frac{1}{2}\mu_0 A_0 c\hat{k}$

$$\vec{B}=\vec{\nabla}\times\vec{A}=\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & 0 & A_z\end{vmatrix}=\hat{x}\frac{\partial A_z}{\partial y}-\hat{y}\frac{\partial A_z}{\partial x}\Rightarrow \vec{B}=\frac{1}{2}\mu_0 A_0\hat{j}$$

Q107. The electric field of a plane wave in a conducting medium is given by

$$\vec{E}(z,t)=\hat{i}E_0 e^{-z/3a}\cos\left(\frac{z}{\sqrt{3}a}-\omega t\right),$$

where ω is the angular frequency and $a>0$ is a constant. The phase difference between the magnetic field $\vec{B}$ and the electric field $\vec{E}$ is

(a) 30^0 and $\vec{B}$ lags behind $\vec{B}$

(b) 30^0 and $\vec{B}$ lags behind $\vec{E}$

(c) 60^0 and $\vec{E}$ lags behind $\vec{B}$

(d) 60^0 and $\vec{B}$ lags behind $\vec{E}$

Ans. : (b)

Solution: $\vec{E}(z,t)=\hat{i}E_0e^{-\kappa z}\cos(kz-\omega t+\delta_E)$ and $\vec{B}(z,t)=\hat{j}B_0e^{-\kappa z}\cos(kz-\omega t+\delta_E+\phi)$

where $\phi=\tan^{-1}\left(\frac{\kappa}{k}\right)$.

$\because \vec{E}(z,t)=\hat{i}E_0e^{-z/3a}\cos\left(\frac{z}{\sqrt{3}a}-\omega t\right)\Rightarrow \kappa=\frac{1}{3a}$ and $k=\frac{1}{\sqrt{3}a}$

$\Rightarrow \phi=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=30^0$

Q108. A hollow waveguide supports transverse electric (TE) modes with the dispersion relation $k=\frac{1}{c}\sqrt{\omega^2-\omega_{mn}^2}$, where ω_{mn} is the mode frequency. The speed of flow of electromagnetic energy at the mode frequency is

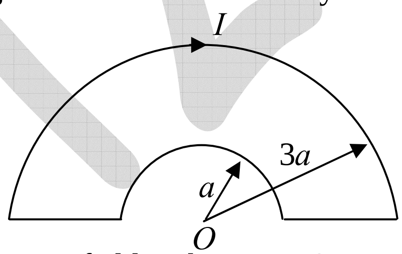
(a) c (b) ω_{mn}/k (c) 0 (d) ∞

Ans. : (c)

Solution: Energy carried by the wave travels at the group velocity

$$v_g=\frac{d\omega}{dk}=c\sqrt{1-\left(\frac{\omega_{mn}}{\omega}\right)^2} \quad \text{at } \omega=\omega_{mn}, v_g=0$$

Q109. The loop shown in the figure below carries a steady current I.



The magnitude of the magnetic field at the point O is

(a) $\frac{\mu_0 I}{2a}$ (b) $\frac{\mu_0 I}{6a}$ (c) $\frac{\mu_0 I}{4a}$ (d) $\frac{\mu_0 I}{3a}$

Ans. : (b)

Solution: $B_a=\frac{1}{2}\frac{\mu_0 I}{2a}\odot$, $B_{3a}=\frac{1}{2}\frac{\mu_0 I}{2(3a)}\otimes$

$$B=B_a-B_{3a}=\frac{\mu_0 I}{4a}\left(1-\frac{1}{3}\right)=\frac{\mu_0 I}{6a}$$

Q110. In the region far from a source, the time dependent electric field at a point (r,θ,ϕ) is

$$\vec{E}(r,\theta,\phi)=\hat{\phi}E_0\omega^2\left(\frac{\sin\theta}{r}\right)\cos\left[\omega\left(t-\frac{r}{c}\right)\right]$$

where ω is angular frequency of the source. The total power radiated (averaged over a cycle) is

(a) $\frac{2\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$ (b) $\frac{4\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$ (c) $\frac{4}{3\pi}\frac{E_0^2\omega^4}{\mu_0 c}$ (d) $\frac{2}{3}\frac{E_0^2\omega^4}{\mu_0 c}$

Ans. : (b)

Solution: $B=\frac{E}{c}$

$$\left|\vec{S}\right|=\frac{1}{\mu_0}E.B=\frac{E^2}{\mu_0 c}=\frac{E_0^2\omega^4}{\mu_0 c}\frac{Sm_\theta^2}{r^2}\cos^2\left[\omega\left(t-\frac{r}{c}\right)\right]$$

$$\left\langle\left|\vec{S}\right|\right\rangle=\frac{1}{2}\frac{E_0^2\omega^4}{\mu_0 c}\frac{\sin^2\theta}{r^2}$$

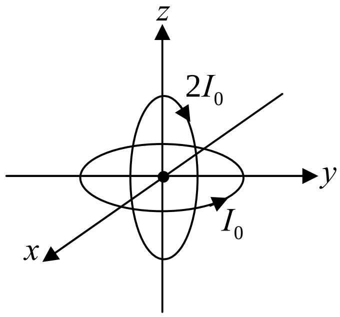
$$P=\oint_S\left\langle\left|\vec{S}\right|\right\rangle.d\vec{a}=\frac{E_0^2\omega^4}{2\mu_0 c}\int_0^{\pi}\int_0^{2\pi}\frac{\sin^2\theta}{r^2}r^2\sin\theta d\theta d\phi$$

$$P=\frac{E_0^2\omega^4}{2\mu_0 c}\times\frac{4}{3}\times 2\pi=\frac{4\pi}{3}\frac{E_0^2\omega^4}{\mu_0 c}$$

NET/JRF (DEC - 2018)

Q111. Two current-carrying circular loops, each of radius R, are placed perpendicular to each other, as shown in the figure.

The loop in the xy- plane carries a current I_0 while that in the xz-plane carries a current $2I_0$. The resulting magnetic field $\vec{B}$ at the origin is



(a) $\frac{\mu_0 I_0}{2R}\left[2\hat{j}+\hat{k}\right]$ (b) $\frac{\mu_0 I_0}{2R}\left[2\hat{j}-\hat{k}\right]$

(c) $\frac{\mu_0 I_0}{2R}\left[-2\hat{j}+\hat{k}\right]$ (d) $\frac{\mu_0 I_0}{2R}\left[-2\hat{j}-\hat{k}\right]$

Ans. : (c)

Solution: Field due to loop in xy plane is $\vec{B}_1 = \frac{\mu_0 I_0}{2R}\hat{z}$

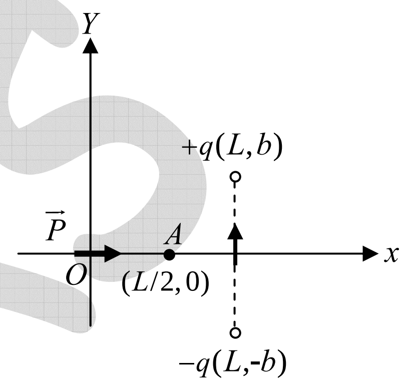
Field due to loop in xz plane is

$$\vec{B}_2 = \frac{\mu_0 (2I_0)}{2R}(-\hat{y})$$

Resultant field $\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I_0}{2R}(-2\hat{y} + \hat{z})$

Q112. An electric dipole of dipole moment $\vec{P} = qb\hat{i}$ is placed at origin in the vicinity of two charges $+q$ and $-q$ at (L, b) and $(L, -b)$, respectively, as shown in the figure.

The electrostatic potential at the point $\left(\frac{L}{2}, 0\right)$ is



(a) $\frac{qb}{\pi\varepsilon_0}\left(\frac{1}{L^2} + \frac{2}{L^2 + 4b^2}\right)$ (b) $\frac{4qbL}{\pi\varepsilon_0\left[L^2 + 4b^2\right]^{3/2}}$

(c) $\frac{qb}{\pi\varepsilon_0 L^2}$ (d) $\frac{3qb}{\pi\varepsilon_0 L^2}$

Ans. : (c)

Solution: Potential due to dipole $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{p\cos 0^0}{(L/2)^2} = \frac{1}{\pi\varepsilon_0}\frac{p}{L^2}$

Potential due to $+q$ charge $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{L^2/4 + b^2}}$

Potential due to $-q$ charge $V_3 = -\frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{L^2/4 + b^2}}$

Resultant $V = V_1 + V_2 + V_3 = \frac{1}{\pi\varepsilon_0}\frac{p}{L^2} \Rightarrow V = \frac{1}{\pi\varepsilon_0}\frac{qb}{L^2}$

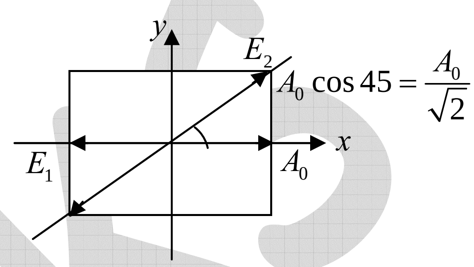
Hence, correct option is (c)

Q113. A monochromatic and linearly polarized light is used in a Young's double slit experiment. A linear polarizer, whose pass axis is at an angle 45^0 to the polarization of the incident wave, is placed in front of one of the slits. If I_{max} and I_{min}, respectively, denote the maximum and minimum intensities of the interference pattern on the screen, the visibility, defined as the ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$, is

(a) $\frac{\sqrt{2}}{3}$ (b) $\frac{2}{3}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\sqrt{\frac{2}{3}}$

Ans. : (b)

Solution: $\vec{E}_1 = \hat{x}A_0 e^{i\omega t}$; $\vec{E}_2 = \frac{A_0}{\sqrt{2}} \frac{(\hat{x}+\hat{y})}{\sqrt{2}} e^{i\omega t + i\delta}$



$$I = \left(\vec{E}_1 + \vec{E}_2\right)\cdot\left(\vec{E}_1^* + \vec{E}_2^*\right)$$

$$\Rightarrow I = \left|\vec{E}_1\right|^2 + \left|\vec{E}_2\right|^2 + \vec{E}_1 \cdot \vec{E}_2^* + \vec{E}_2 \cdot \vec{E}_1^*$$

$$= A_0^2 + \frac{A_0^2}{4} + (1+1) + \frac{A_0^2}{2} e^{-i\delta} + \frac{A_0^2}{2} e^{i\delta}$$

$$\Rightarrow I = A_0^2 + \frac{A_0^2}{2} + \frac{A_0^2}{2} \frac{e^{i\delta} + e^{-i\delta}}{2} = \frac{3A_0^2}{2} + A_0^2 \cos\delta$$

$$I_{max} = \frac{5A_0^2}{2}, I_{min} = \frac{A_0^2}{2} \Rightarrow V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{2}{3}$$

Q114. An electromagnetic wave propagates in a nonmagnetic medium with relative permittivity $\varepsilon = 4$. The magnetic field for this wave is

$$\vec{H}(x, y) = \hat{k}H_0 \cos\left(\omega t - \alpha x - \alpha\sqrt{3}y\right)$$

where H_0 is a constant. The corresponding electric field $\vec{E}(x, y)$ is

(a) $\frac{1}{4}\mu_0 H_0 c\left(-\sqrt{3}\hat{i} + \hat{j}\right)\cos\left(\omega t - \alpha x - \alpha\sqrt{3}y\right)$

(b) $\frac{1}{4}\mu_0 H_0 c\left(\sqrt{3}\hat{i} + \hat{j}\right)\cos\left(\omega t - \alpha x - \alpha\sqrt{3}y\right)$

(c) $\frac{1}{4}\mu_0 H_0 c\left(\sqrt{3}\hat{i} - \hat{j}\right)\cos\left(\omega t - \alpha x - \alpha\sqrt{3}y\right)$

(d) $\frac{1}{4}\mu_0 H_0 c\left(-\sqrt{3}\hat{i} - \hat{j}\right)\cos\left(\omega t - \alpha x - \alpha\sqrt{3}y\right)$

Ans. : (a)

Solution: $\vec{E} = -v\left(\hat{K} \times \hat{B}\right)$

$$\vec{K} = \alpha\hat{x} + \alpha\sqrt{3}\hat{y} \Rightarrow \hat{K} = \frac{\vec{K}}{\left|\vec{K}\right|} = \frac{\alpha\hat{x} + \alpha\sqrt{3}\hat{y}}{\sqrt{\alpha^2 + 3\alpha^2}} = \frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}$$

$$\Rightarrow E = \frac{-c}{\sqrt{\varepsilon_r}}\left[\frac{\hat{x} + \sqrt{3}\hat{y}}{2} \times \mu_0 H_0 \cos\left(\omega t - \alpha x - \alpha\sqrt{3}y\right)\hat{z}\right]$$

$$\Rightarrow E = \frac{-c\mu_0 H_0}{2\sqrt{4}}\left[\left(-\hat{y} + \sqrt{3}\hat{x}\right)\cos\left(\omega t - \alpha x - \sqrt{3}y\right)\right]$$

$$\Rightarrow E = \frac{1}{4}c\mu_0 H_0\left(-\sqrt{3}\hat{x} + \hat{y}\right)\cos\left(\omega t - \alpha x - \alpha\sqrt{3}y\right)$$

Q115. In an inertial frame uniform electric and magnetic field $\vec{E}$ and $\vec{B}$ are perpendicular to each other and satisfy $\left|\vec{E}\right|^2 - \left|\vec{B}\right|^2 = 29$ (in suitable units). In another inertial frame, which moves at a constant velocity with respect to the first frame, the magnetic field is $2\sqrt{5}\hat{k}$. In the second frame, an electric field consistent with the previous observations is

(a) $\frac{7}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$ (b) $7\left(\hat{i} + \hat{k}\right)$ (c) $\frac{7}{\sqrt{2}}\left(\hat{i} + \hat{k}\right)$ (d) $7\left(\hat{i} + \hat{j}\right)$

Ans. : (a)

Solution: $\left|\vec{E}\right|^2 - \left|\vec{B}\right|^2 = 29$

In another Frame $\left|\vec{E}'\right|^2 - \left|\vec{B}'\right|^2 = 29$

$$\vec{B}' = 2\sqrt{5}\hat{k} \Rightarrow \left|B'\right|^2 = 4 \times 5 = 20 \Rightarrow \left|\vec{E}'\right|^2 = 49$$

It is given $\vec{E} \perp \vec{B}$ so $\vec{E}' = \frac{7}{\sqrt{2}}\left(\hat{i} + \hat{j}\right)$

Q116. Electromagnetic wave of angular frequency ω is propagating in a medium in which, over a band of frequencies the refractive index is $n(\omega) \approx 1 - \left(\frac{\omega}{\omega_0}\right)^2$, where ω_0 is a constant. The ratio $\frac{v_g}{v_p}$ of the group velocity to the phase velocity at $\omega = \frac{\omega_0}{2}$ is

(a) 3 (b) $\frac{1}{4}$ (c) $\frac{2}{3}$ (d) 2

Ans. : (a)

Solution: $n = 1 - \frac{\omega^2}{\omega_0^2}$

$$n = \frac{c}{v_p} = 1 - \frac{\omega_0^2/4}{\omega_0^2} = \frac{3}{4} \Rightarrow v_p = \frac{4c}{3}$$

$$n = \frac{ck}{\omega} = 1 - \frac{\omega^2}{\omega_0^2} \Rightarrow kc = \omega - \frac{\omega^3}{\omega_0^2}$$

$$\Rightarrow \frac{dk}{d\omega}.c = 1 - \frac{3\omega^2}{\omega_0^2} = 1 - 3\frac{\omega_0^2/4}{\omega_0^2} = \frac{1-3}{4} = \frac{1}{4} \Rightarrow v_g = \frac{d\omega}{dk} = 4c$$

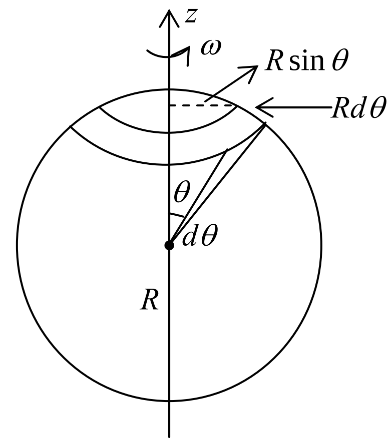
Thus, $\frac{v_g}{v_p} = \frac{4c}{4c/3} = 3$

Q117. A rotating spherical shell of uniform surface charge and mass density has total mass M and charge Q. If its angular momentum is L and magnetic moment is μ, then the ratio $\frac{\mu}{L}$ is

(a) $\frac{Q}{3M}$ (b) $\frac{2Q}{3M}$ (c) $\frac{Q}{2M}$ (d) $\frac{3Q}{4M}$

Ans. : (c)

Solution: $I = \frac{2}{3}MR^2, L = I\omega = \frac{2}{3}MR^2\omega$



$$\sigma = \frac{Q}{4\pi R^2}$$

$$dm = dI \times \pi (R\sin\theta)^2 = \frac{\sigma \times (2\pi R\sin\theta)(Rd\theta) \times \pi R^2 \sin^2\theta}{2\pi/\omega}$$

$$dm = \pi\sigma\omega R^4 \sin^3\theta d\theta$$

$$\mu = \int dm = \frac{4}{3}\pi\sigma\omega R^4 = \frac{4}{3}\pi\left(\frac{\theta}{4\pi R^2}\right)\omega R^4 \Rightarrow \mu = \frac{QR^2\omega}{3}$$

$$\frac{\mu}{L} = \frac{QR^2\omega/3}{\frac{2}{3}MR^2\omega} = \frac{QR^2\omega}{3} \times \frac{3}{2MR^2\omega} = \frac{Q}{2M}$$