K24U 1619

Reg. No.: ....

Name : ....

Second Semester B.Sc. Degree (CBCSS – OBE-Regular/Supplementary/ Improvement) Examination, April 2024
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
2B02 MAT : Integral Calculus and Logic

Time : 3 Hours Max. Marks : 48

UNIT – I

Short answer type. Answer **any 4** questions. **Each** question carries **1** mark. **(4×1=4)**

1. Define hyperbolic cosine of x.
2. Write the equation of the circle of radius |a| centered at O in polar co-ordinates.
3. Find the Cartesian equivalent of the Polar equation $r \cos \theta = 2$.
4. Define a statement.
5. What you mean by a contingency ?

UNIT – II

Short essay type. Answer **any 8** questions. **Each** question carries **2** marks. **(8×2=16)**

6. Prove that $\cosh^2 x - \sinh^2 x = 1$.
7. Integrate log x.
8. Find the Cartesian equivalent to the polar equation $r \cos\left(\theta - \frac{\pi}{4}\right) = \sqrt{2}$.
9. Evaluate $I = \int_0^1 \int_0^2 xy(x - y)\,dx\,dy$.
10. Find the area bounded between the curve $y = x^2$ above the x-axis and below the line y = 2.

P.T.O.

11. Define the error of approximation.

12. Write the formula using in Simpson's 1/3 rule of integration.

13. Find the conjunction of the propositions p and q where p is the proposition "Today is Friday" and q is the proposition "It is raining today".

14. Let $a \geq 0$ be a real number. If for every $\varepsilon > 0$, we have $0 \leq a < \varepsilon$, then prove that $a = 0$.

15. Prove that the square of an odd integer is also an odd integer.

16. Examine that the following argument is valid : $p, p \rightarrow q \vdash q$.

UNIT – III

Essay type. Answer **any 4** questions. **Each** question carries **4** marks. **(4×4=16)**

17. Evaluate $\int \coth 5x dx$.

18. Show that $\int \frac{\sin^4 x}{\cos^2 x} dx = \frac{\sin^3 x}{\cos x} + \frac{3}{2} \sin x \cos x - \frac{3}{2} x$.

19. Evaluate $\iint_S (x^2 + y^2) dx\, dy$ over the region S in which $x \geq 0$; $y \geq 0$ and $x + y \leq 1$.

20. Find the volume of $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

21. Evaluate $\int_0^2 \frac{dx}{x^2 + 2x + 10}$. Using Simpson's rule with n = 2, 4. Compare with the exact solutions.

22. Show that $\neg(p \vee q)$ and $\neg p \wedge \neg q$ are logically equivalent.

23. Show that the hypothesis " If you send me an e-mail message, then I will finish writing the program," " If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

UNIT – IV

Long essay type. Answer **any 2** questions. **Each** question carries **6** marks. **(2×6=12)**

24. If $U_n = \int_0^{\pi/2} \theta \sin^n \theta d\theta$ and $n > 1$, prove that $U_n = \frac{1}{n^2} + \frac{n-1}{n} U_{n-2}$. Deduce that $U_5 = \frac{149}{225}$.

25. Use triple integration in cylindrical coordinates to find the volume and the centroid of the solid G that is bounded above by the hemisphere $z = \sqrt{25 - x^2 - y^2}$, below by the xy-plane, and laterally by the cylinder $x^2 + y^2 = 9$.

26. Evaluate $\int_0^1 \frac{dx}{3+2x}$, using trapezoidal rule with $n = 2, 4$. Compare with the exact solution. Find the bound on the error. Also, find the number of sub-intervals required if the error is to be less than 5×10^{-4}.

27. Prove that the following argument is valid : $p \to \neg q$, $r \to q$, $r \neg p$.