Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (C.B.C.S.S.– O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2022
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
1B01 MAT : Set Theory, Differential Calculus and Numerical Methods

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **four** questions from this Part. **Each** question carries **one** mark.

1. Define a relation on set of integers.

2. Find the nth derivative of $x^3 + 5x^2 + 3$.

3. Find the domain of log x.

4. State Euler's theorem on homogeneous functions.

5. Let $w = x^2y + 2y$. Find $\dfrac{\partial w}{\partial x}$ and $\dfrac{\partial w}{\partial y}$.

PART – B

Answer **any eight** questions from this Part. **Each** question carries **two** marks.

6. Define reflexive relation on a set and give an example.

7. Show that the relation $\leq$, is a partial relation on set of all real numbers.

8. Give an example of a function which is one-one, but not on-to.

9. Give an example of algebraic and transcendental equation.

10. Find $\lim\limits_{x \to 0} f(x)$, where $f(x) = \begin{cases} 0 & \text{if } x \geq 0 \\ 1 & \text{if } x < 0 \end{cases}$.

P.T.O.

11. Show that $\lim\limits_{\theta \to 0} \sin \theta = 0$.

12. For what values of a is $f(x) = \begin{cases} a^2x - 2a, \text{ if } x \geq 2 \\ 12 \text{ if } x \leq 2 \end{cases}$ continuous at every x ?

13. Describe the domain of the function $f(x, y) = \sqrt{y - x^2}$.

14. Find the values of $\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}$ at the point (4, –5) if f(x, y) = x² + 3xy + y – 1.

15. Let f(x, y) = 2x + 3y – 4. Find the slope of the line tangent to this surface at the point (2, –1) and lying in the plane x = 2.

16. Find $\dfrac{dy}{dx}$ using implicit differentiation, if y² – x² – sin xy = 0.

PART – C

Answer **any four** questions from this Part. **Each** question carries **four** marks.

17. Show that (A × B) ∩ (A × C) = A × (B ∩ C).

18. Let A = {a, b} and B = {1, 2, 3}

    i) Find number of functions from A to B

    ii) Find number of functions from B to A.

19. Determine the maximum number of positive and negative roots and intervals of length one unit in which the real roots lie for the following equation
    8x³ – 12x² – 2x + 3 = 0.

20. If ax² + 2hxy + by² = 1, then show that $\dfrac{d^2y}{dx^2} = \dfrac{h^2 - ab}{(hx + by)^3}$.

21. Evaluate $\lim\limits_{x \to 0} \dfrac{\sqrt{x^2 + 100} - 10}{x^2}$.

22. Describe the level surfaces of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.

23. State mixed partial theorem. Verify it for the function w = x sin y + y sin x + xy.

PART – D

Answer **any two** questions from this Part. **Each** question carries **six** marks.

24. i) Let $f : R \to R$, defined by $f(x) = 2x - 3$, find the formula for f^{-1}.

    ii) Consider the function $f : A \to B$, $g : B \to C$, then prove that if g of is one-one, then f is one-one.

25. Find the root correct to two decimal places of the equation $f(x) = xe^x - \cos x = 0$, using the method of false position.

26. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, then show that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

27. i) Find $\lim\limits_{(x, y) \to (0, 0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

    ii) At what points (x, y) in the plane are the function $f(x, y) = \sin\dfrac{1}{xy}$ is continuous.

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