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Contents

OPERATORS IN SCHATTEN CLASSES AND K-FRAMES

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Abstract: *In this paper we prove some results on K -frames. Also we study Schatten class operators in terms of K -frames.*

Introduction

R.J. Duffin and A.C.Schaffer introduced frames in Hilbert spaces while working on nonharmonic Fourier series. Later, Daubechies, Grossmann and Meyer gave a strong place to frames in harmonic analysis. Frame theory plays an important role in signal processing, sampling theory, coding and communications and so on. Any vector in a separable Hilbert space can be expressed uniquely in terms of an orthonormal basis of the Hilbert space. Difficulty in finding an orthonormal basis for a separable Hilbert space and the uniqueness of the expression of vectors, in terms of an orthonormal basis makes orthonormal bases undesirable in applications. In order to overcome this strain practitioners brought frames as a replacement to orthonormal bases.

Notion of K -frames were introduced by L.Găvruta, to study atomic systems with respect to bounded linear operators. K -frames are more general than classical frames, in the sense that the lower bound only holds for the elements in the range of K .

In section 2 we record some of the basic definitions and results in frame theory and K -frames.In section 3 we have included some results on K-frames. Section 4 contains our main results relating K -frames and operators in Schatten classes.

Throughout this paper. H is a separable Hilbert space and we denote by $B(H)$,the space of all linear bounded operators on H .For $K \in B(H)$,we denote $R(K)$ the range of K .Also, $GL(H)$ denote the set of all bounded linear operators which have bounded inverses

1. Preliminaries

In this section we give definitions and some basic results in frames and K-frames which are required in the sequel. [2] is a good reference for basics of frame theory.

Definition 1.1[2]

A *frame* in a separable Hilbert space H is a countable family $\{f_k : k \in N\}$ of vectors for which, there are positive constants A and B, satisfying the inequalities,

$$A\|x\|^2 \le \sum_{k=1}^{\infty} |\langle f, f_k \rangle|^2 \le B\|x\|^2, \forall x \in H.$$

In particular, if $A = B$, we say that $\{f_k : k \in N\}$ is a tight frame in H.

Definition 1.2[3]

Let $K \in B(H)$. We say that $\{f_n\}$ in H is a K-frame for H if there exist constants $A, B > 0$ such that

$$A\|K^* x\|^2 \le \sum_{n=1}^{\infty} |\langle x, f_n \rangle|^2 \le B\|x\|^2, \text{ for all } x \in H.$$

Definition 1.3[3]

Let $K \in B(H)$. We say that $\{f_n\}$ is an atomic system for K if the following statements hold.

(i) the series $\sum c_n f_n$ converges for all $c = (c_n) \in l^2$.

(ii) there exists $c > 0$ such that for every $x \in H$ there exists $a_x = (a_n) \in l^2$ such that

$$\|a_x\|_{l^2} \le C\|x\| \text{ and } Kx = \sum_n a_n f_n.$$

Following theorem gives a characterization of K -frames.

Theorem 1.4[3]

Let $\{f_n\} \subset H$. Then the following are equivalent

(i) $\{f_n\}$ is an atomic system for K.

(ii) There exists $A, B > 0$ such that

$$A\|K^* x\|^2 \le \sum_{n=1}^{\infty} |\langle x, f_n \rangle|^2 \le B\|x\|^2, \text{ for any } x \in H$$

(iii) $\{f_n\}$ is a Bessel sequence and there exists a Bessel sequence $\{g_n\}$ such that

$$Kx = \sum_{n=1}^{\infty} \langle x, g_n \rangle f_n \text{ for all } x \in H.$$

(iv) $\sum_{n=1}^{\infty} |\langle x, f_n \rangle|^2 < \infty$ and there exists a Bessel sequence $\{g_n\}$ such that

$$K^* x = \sum_{n=1}^{\infty} \langle x, f_n \rangle g_n \quad \text{for all } x \in H.$$

Now we give some of the earlier results and definitions which are required for our purposes.

Theorem 1.5[5]

Let $T \in B(H)$ and $\{f_n\}$ be a K -frame for H .Then $\{Tf_n\}$ is a TK -frame for H

Theorem 1.6

$T \in B(H)$ is an injective and closed range operator if and only if there exists a constant $c > 0$ such that $c\|x\|^2 \leq \|Tx\|^2$ for all $x \in H$.

Theorem 1.7[5]

If $\{f_n\}$ is an ordinary frame for H ,then $\{Kf_n\}$ is a K -frame for H.

Theorem 1.8[5]

If $\{f_n\}$ is a K -frame for H ,then $\{K^N f_n\}$ is a K^N -frame for H where $N \geq 1$ is a fixed integer.

Theorem 1.9[4]

$\mathcal{F}_K(H) \subset \mathcal{F}_M(H)$ if and only if $R(K) \supset R(M)$ where $\mathcal{F}_K(H), \mathcal{F}_M(H)$ denote the set of all K -frames and M -frames on H.

In the following two sections we give our main results.

2. K-Frames

In this section we present two results on K -frames.

Theorem 2.1

If $\{f_n\}$ is a frame for $R(K)$, then $\{K^* f_n\}$ is a K^*-frame for H.

Proof:

Let $\{f_n\}$ be a frame for $R(K)$. Then there exist constants $A, B > 0$ such that

$$A\|x\|^2 \leq \sum_{n=1}^{\infty} |\langle x, f_n \rangle|^2 \leq B\|x\|^2 \text{ for all } x \in R(K).$$

Therefore, for all $x \in H$,

$$A\|Kx\|^2 \leq \sum_{n=1}^{\infty} |\langle Kx, f_n \rangle|^2 \leq B\|Kx\|^2 .$$

$$A\|(K^*)^* x\|^2 \leq \sum_{n=1}^{\infty} |\langle x, K^* f_n \rangle|^2 \leq B\alpha^2 \|x\|^2 \text{ for all } x \in H$$

and for some $\alpha > 0$. Hence $\{K^* f_n\}$ is a K^* -frame for H .

Theorem 2.2

Let $K \in B(H)$ be an injective and closed range operator.If $\{f_n\}$ is a frame for $R(K)$, then $\{KK^* f_n\}$ is a K-frame for H.

Proof:

Let $\{f_n\}$ be a frame for $R(K)$. Then there exist constants $A, B > 0$ such that

$$A\|x\|^2 \leq \sum_{n=1}^{\infty} |\langle x, f_n \rangle|^2 \leq B\|x\|^2 \text{ for all } x \in R(K).$$

Also, by our assumption, there exists $c > 0$ such that $c\|x\|^2 \leq \|Kx\|^2$ for all $x \in H$.

For $x \in H, Kx \in R(K)$, and we get

$$A\|Kx\|^2 \leq \sum_{n=1}^{\infty} |\langle Kx, f_n \rangle|^2 \leq B\|Kx\|^2$$

Therefore

$$Ac\|x\|^2 \leq A\|Kx\|^2 \leq \sum_{n=1}^{\infty} |\langle Kx, f_n \rangle|^2 \leq B\|Kx\|^2 \leq B\alpha^2 \|x\|^2$$

for all $x \in H$ and for some $\alpha > 0$. That is,

$$Ac\|x\|^2 \leq \sum_{n=1}^{\infty} |\langle Kx, f_n \rangle|^2 \leq B\alpha^2 \|x\|^2 \text{ for all } x \in H .$$

$$i.e.\ E\|x\|^2 \le \sum_{n=1}^{\infty} \left|\langle x, K^* f_n\rangle\right|^2 \le F\|x\|^2 \text{ for all } x \in H$$

Where $E = Ac > 0, F = B\alpha^2 > 0$. Therefore $\{K^* f_n\}$ is a frame for H and hence $\{KK^* f_n\}$ is a K-frame for H.

3. Some results relating K-frames and operators in Schatten classes

Definition 3.1[6]

Let T be a compact operator on a separable Hilbert space H. Given $0 < p < \infty$, we define the **Schatten p-class** of H, denoted by $S_p(H)$ or simply S_p, to be the space of all compact operators T on H with its singular value sequence $\{\lambda_n\}$ belonging to l^p.

$S_p(H)$ is a two sided ideal in $B(H)$.

Following two theorems by H.Bingyang, L.H.Koi and K.Zhu gives a characterization for Schatten p- class operators in terms of frames.

Theorem3.1 [1]

Suppose T is a compact operator on H and $2 \le p < \infty$. Then the following conditions are equivalent.

(a) $T \in S_p$.

(b) $\{\|Te_n\|\} \in l^p$ for every orthonormal basis $\{e_n\}$ in H.

(c) $\{\|Tf_n\|\} \in l^p$ for every frame $\{f_n\}$ in H.

Theorem 3.2 [1]

Suppose T is a compact operator on H and $0 < p \le 2$. Then the following conditions are equivalent.

(a) $T \in S_p$.

(b) $\{\|Te_n\|\} \in l^p$ for some orthonormal basis $\{e_n\}$ in H.

(c) $\{\|Tf_n\|\} \in l^p$ for some frame $\{f_n\}$ in H.

Theorem 3.3

Suppose T is a compact operator on H and $2 \leq p < \infty$ .If T is in the Schatten class S_p, then $\{\|Tf_n\|\} \in l^p$ for every K -frame $\{f_n\}$ in H.

Proof:

Suppose $\in S_p, 2 \leq p < \infty$.

Let $\{f_n\}$ be a K -frame for H and $\{e_n\}$ be an orthonormal basis for H .

Then $\{h_n\} = \{f_n\} \cup \{e_n\}$ is a frame for H and $\{\|Th_n\|\} \in l^p, 2 \leq p < \infty$.

Therefore $\{\|Tf_n\|\} \in l^p$, $2 \leq p < \infty$ and the result is proved.

Theorem 3.4

Suppose T is a compact operator on H and $2 \leq p < \infty$ .If $\{Tf_n\} \in l^p$ for every K-frame $\{f_n\}$ in H ,then $\{TKe_n\} \in l^p$ for every orthonormal basis $\{e_n\}$ in H.

Proof:

Let $\{e_n\}$ be an orthonormal basis for H .Then $\{Ke_n\}$ is a K -frame for H.

Therefore by our assumption $\{\|TKe_n\|\} \in l^p, 2 \leq p < \infty$. Hence $\{\|TKe_n\|\} \in l^p$ for every orthonormal basis $\{e_n\}$ in H.

Theorem 3.5

Suppose T is a compact operator on H and $K \in GL(H)$ and $2 \leq p < \infty$ .Then the following are equivalent.

(a) T is in the Schatten class S_p.

(b) $\{\|Tf_n\|\} \in l^p$ for every K -frame $\{f_n\}$ in H .

Proof:

Suppose (b) holds.

Then $\{\|TKe_n\|\} \in l^p$ for every orthonormal basis $\{e_n\}$ in H . This implies that $TK \in S_p$.

Using the fact that S_p is a two-sided ideal in $B(H)$, $TKK^{-1} = T \in S_p$. This completes the proof.

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Preface

This proceedings contains the substance of invited lectures and contributed research presentations delivered at the Two day National Conference on Algebra, Analysis and Related Topics held on 16th and 17th February 2018 at Payyanur College, Payyanur. The conference was organized by the Department of Mathematics, Payyanur College and was sponsored jointly by the Kerala State Council for Science, Technology and Environment and the PTA Payyanur College, Payyanur.

The seminar was organised in honour of Dr. K.T. Ravindran, on the occasion of his retirement. Dr. K.T. Ravindran was the principal of the college during 2016-18 and an associated Professor and research guide in the Department of Mathematics for several years.

The objective of the conference was to bring together the experts as well as young researchers working in the areas of Analysis and Algebra for an exchange of ideas and to provide graduate students and post graduate students an exposure to the cutting edge developments in this field. There were 8 invited lectures each on topics of current active research in analysis or algebra. The conference was attended by more than 200 participants from various colleges and universities inside and outside the state.

We would like to thank all those who participated in the conference including the speakers and the persons who chaired various sessions. Thanks are also due to the College Management, PTA, Dr. K.T. Ravindran, Principal, Payyanur College and Dr. Harikrishnan P. K, Associate Professor, Dept. of Mathematics, MIT, Manipal for their help and support.

Finally I express my deep sense of gratitude to my colleagues in the Department of Mathematics and my dear students for their hard work and whole-hearted co-operation which made the conference a real success.

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Contents

Conclusion

In this paper we initiated a study on $L(h,k)$-set labeling of graphs $L(h,1)$-set labeling number of some classes of graphs is obtained. $L(h,k)$-set indexers of graphs has been defined and obtained bounds for $L(2,1)$-set indexers of graphs. It is possible to investigate similar results to other graph families. Certain problems in this area are still open.

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