K20U 0127

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2020
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B10MAT : Linear Algebra

Time : 3 Hours | Max. Marks : 46

SECTION – A

All the first 4 questions are **compulsory.** They carry **1** mark **each.**

1. Give an example to show that if f and g are two quadratic polynomials then the polynomial f + g need not be quadratic.

2. Obtain a basis for $M_{2\times 2}(R)$.

3. Let $V = P_2(R)$ and let $\beta = \{1, x, x^2\}$ be the standard ordered basis for V. If $f(x) = 3x^2 + 2x + 1$ then $[f]_\beta$ is

4. Give the nature of characteristic roots of
   i) a Hermitian matrix and
   ii) a Unitary matrix.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each.**

5. Find the equation of the line through the points P (2, 0, 1) and Q (4, 5, 3).

6. What is the possible difference between a generating set and a basis ?

7. Is the union of two subspaces W_1 and W_2 of a vectorspace V again a subspace of V ? Justify with an example.

P.T.O.

8. Let V be a vectorspace and $\beta = \{x_1, x_2, ..., x_n\}$ be a subset of V. Show that β is basis if each vector y in V can be uniquely expressed as a linear combination of vectors in β.

9. Show that $T : R^2 \to R^2$ defined by $T(a_1, a_2) = (2a_1 + a_2, a_1)$ is a linear transformation.

10. Let $T : V \to W$ be a linear transformation. Prove that N (T), the nullspace of T, is a subspace of V.

11. Find a basis of the row space of the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 5 \\ 2 & 3 & 4 \end{bmatrix}.$$

12. Find the characteristic values of the matrix

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}.$$

13. Use Gauss elimination to solve the system of equations :

$10x + y + z = 12$

$2x + 10y + z = 13$

$x + y + 3z = 5.$

14. Use Gauss, Jordan elimination to solve the system of equations :

$10x + y + z = 12$

$2x + 10y + z = 13$

$x + y + 3z = 5.$

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. In every vectorspace V over a field F prove that

    i) $a0 = 0 \ \forall a \in F$, where 0 is the zero vector and

    ii) $(-a)x = -(ax) \ \forall a \in F$ and $\forall x \in V$.

16. Define linear dependence and linear independence of vectors with examples.

17. Define a linear transformation from a vectorspace V into W. Verify that $T : M_{m \times n} \to M_{n \times m}$ by $T(A) = A^t$ where A^t is the transpose of A, is linear.

18. Show that the row nullity and column nullity of a square matrix are equal.

19. Find the characteristic values and the corresponding characteristic vectors of the matrix.

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

20. Use the Gaussian elimination method to find the inverse of the matrix.

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. If a vectorspace V is generated by a finite set S_0, then show that a subset of S_0 is a basis for V and V has a finite basis.

22. State and prove dimension theorem. Deduce that a linear transformation $T : V \to V$ is one to one if and only if T is onto.

23. Show that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

satisfies Cayley Hamilton theorem and hence obtain A^{-1}.

24. Prove that

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

is diagonalizable and find the diagonal form.

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VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2020
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B11MAT : Numerical Methods and Partial Differential Equations

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **4** questions are **compulsory**. They carry **1** mark **each**.

1. State the intermediate value theorem for finding the real root of an equation.

2. Complete the expression $\Delta = E -$

3. Give the maximum bound for error $R_1(f)$ in trapezoidal rule.

4. For a function $u(r, \theta, t)$, give its Laplacian in Polar co-ordinates.

SECTION – B

Answer **any 8** questions from among the questions **5 to 14**. These questions carry **2** marks **each**.

5. Find $\sqrt{15}$ by Bisection method, correct to two decimal places.

6. Find a root of the equation log x – cos x = 0, where x is in radians, correct to two decimal places, using Regula Falsi method.

7. Show that $\Delta\left(\frac{f_i}{g_i}\right) = \frac{g_i\Delta f_i - f_i\Delta g_i}{g_i g_{i+1}}$.

P.T.O.

8. Find $\log_e(2.7)$ from the following table using Lagranges interpolation formula.

x	2	2.5	3
$\log_e(x)$	0.6932	0.9163	1.0986

9. Evaluate $\frac{2}{\sqrt{\pi}}\int_0^1 e^{-x^2}dx$, using Simpson's 1/3 rule, taking h = 0.25.

10. Evaluate $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal rule with h = 0.25.

11. Find a solution to the initial value problem $y' = 2y - x$, $y(0) = 1$, by performing two iterations of the Picard's method.

12. Find y(1.2), given the differential equation $y' = -2xy^2$, with the condition y(1) = 1, using Taylor's series with step size h = 0.1.

13. Give the Fourier series solution of the one dimensional wave equation, with fixed ends and initial conditions $u(x, 0) = f(x)$ and $\left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x)$.

14. Solve the equation $u_{yy} = 0$ where u is a function of x and y.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Find a real root of the equation $x^3 + x^2 - 1 = 0$ by General iteration method, correct to two decimal places.

16. Using Newtons divided difference formula, find a cubic polynomial for the following data. Hence find f(3).

x	0	1	2	4
f(x)	1	1	2	5

17. The function f(x) represented by the following data has a minimum in the interval (0.5, 0.8). Find this point of minimum and the minimum value.

x	0.5	0.6	0.7	0.8
f(x)	1.3254	1.1532	0.9432	1.0514

18. Find the approximate value of $y(0.1)$ given that $y' = x^2 + y^2$, $y(0) = 1$ using three iterations of the Modified Euler's method with $h = 0.1$.

19. Given $\frac{dy}{dx} = y - x$ with $y(0) = 2$, use Runge – Kutta method of order two to find $y(0.2)$ taking $h = 0.1$.

20. A stretched string of length l and fixed end points has initial displacement $y = a \sin \frac{\pi x}{l}$ from which it is released at time $t = 0$. Find the vertical displacement $y(x, t)$ at any distance x from one end at time t.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Find an interval of unit length which contains the smallest positive root of the equation $e^x - 2x^2 = 0$. Hence find the root of this equation using Newton – Raphson method correct to three decimal places.

22. The following table gives the value of e^{-x} for some values of x :

x	0.2	0.3	0.4	0.5	0.6	0.7	0.8
e^{-x}	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493

Determine the value of $e^{-0.55}$ using Stirling's central difference formula.

23. Compute $f'(0.2)$ and $f''(0)$ from the following table.

x	0.0	0.2	0.4	0.6	0.8	1.0
f(x)	1.00	1.16	3.56	13.96	41.96	101.00

24. Find the temperature $u(x, t)$ in a slab of length L whose ends are kept at zero temperature and whose initial temperature $f(x)$ is given by

$$f(x) = \begin{cases} k, & \text{when } 0 < x < \frac{L}{2} \\ 0, & \text{when } \frac{L}{2} < x < L \end{cases}$$

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K20U 0129

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.) Examination, April 2020
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B12MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Sketch the region $\{z : Re\,(iz) \geq 0\}$.

2. Define Harmonic function.

3. Find the Radius of convergence of $\sum 7^n z^n$.

4. Find the residue of $f(z) = e^z$ at $z = 0$.

SECTION – B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Give an example of a function which is differentiable exactly at one point and give its justification.

6. Verify Cauchy-Riemann equations for the function $f(z) = z^2$.

7. Evaluate $\int_C |z|\, dz$, where C is the line segment from origin to 1 + i.

8. Find the Radius of convergence of $\sum (1 + i)^n (z - 3i)^n$.

9. Find the residue of $f(z) = \dfrac{9z+i}{z(z^2+1)}$ at $z = i$.

P.T.O.

10. Find the Laurent's series expansion of $f(z) = \frac{1}{z^5} \sin z$ with center 0.

11. State Taylors Theorem. Find the Taylors series expansion of $f(z) = \frac{1}{1+z^2}$ centered at $z = 0$.

12. Give an example of a series which is convergent but not absolutely. Give justification.

13. State Laplace's Equation. Give an example of a real valued function which satisfy Laplace's Equation on the complex plane.

14. State Cauchy's inequality.

SECTION – C

Answer **any 4** questions from among the questions **15 to 20**. These questions carry **4** marks **each.**

15. Prove that an analytic function of constant absolute value is constant in a domain.

16. Evaluate the following :

    a) $\int_{0}^{1-i} z^2 dz$

    b) $\int_{8+\pi i}^{8-3\pi i} e^{z/2} dz$

17. The power series $\Sigma a_n z^n$ converge at $z = 1$ and diverge at $z = -1$. Find the radius of convergence of $\Sigma a_n z^n$.

18. State and prove Residue Theorem.

19. Find an analytic function $f(z) = u(x, y) + iv(x, y)$, where $u(x, y) = xy$.

20. State and prove the theorem of convergence of power series.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. State and prove Cauchy – Riemann equations.

22. a) Define singular point, isolated singular point, removable singular point, pole and essential singular point.

    b) Give an example of a non-isolated singular point.

23. a) State and prove Cauchy's integral formula.

    b) Evaluate $\int_C \frac{e^z}{z-2}\,dz$, where C is the circle $|z| = 3$.

24. Give examples and justifications of power series having Radius of convergence 1 and

    a) Which diverge at every point on the circle of convergence ?

    b) Which doesn't diverge at every point on the circle of convergence ?

____

K20U 0130

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2020
(2014 Admission Onwards)
Core Course in Mathematics
6B13MAT : MATHEMATICAL ANALYSIS AND TOPOLOGY

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. If $I = [0, 4]$, calculate the norm of the partition $\mathcal{P} = \{0, 1, 1.5, 2, 3.4, 4\}$.

2. Evaluate $\lim (f_n(x))$ where $f_n(x) = \frac{x}{x-n}$ for all $x \geq 0$, $n \in \mathbb{N}$.

3. Fill in the blanks : The closure of set of all irrational numbers is ________

4. Write a pair of topologies T_1 and T_2 on $X = \{a, b, c\}$ so that $T_1 \cup T_2$ is not a topology on X.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each.**

5. Show that every constant real valued function on [a, b] is in $\mathcal{R}$ [a, b].

6. State squeeze theorem for Riemann integrability.

7. Find the value of $\int_{-1}^{10} \text{sgn}(x)\,dx$.

8. Prove that the sequence of functions, $f_n(x) = \frac{x}{n}$; $n \in \mathbb{N}$ converges uniformly on [0, 1]

9. State the Bounded Convergence Theorem.

10. Define a metric space and write an example.

11. Prove that in a metric space each open sphere is an open set.

12. Give an example of a pair of subsets A and B of the real line with usual topology such that Int (A) $\cup$ Int(B) $\neq$ Int (A $\cup$ B).

13. Define subspace of a topological space and show that it is a topological space.

14. Is the real line $\mathbb{R}$ with the usual topology separable ? Justify.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**.

15. Prove that if $f : [a, b] \to \mathbb{R}$ is continuous on [a, b], then $f \in \mathcal{R}[a, b]$.

16. State and prove composition theorem in Riemann integrals. Deduce that if $f \in \mathcal{R}[a, b]$, then $|f| \in \mathcal{R}[a, b]$ and $\left|\int_a^b f\right| \le \int_a^b |f|$.

17. Prove that a power series $\sum a_n x^n$ is absolutely convergent if $|x| < R$ and is divergent if $|x| > R$. (Here R is the radius of convergence and assume that $0 < R < \infty$).

18. Show that a subset F of a metric space is closed if and only if its complement F' is open.

19. Prove that a subspace Y of a complete metric space X is complete if and only if it is closed.

20. Prove that in a topological space $\overline{A} = A \cup D(A)$ and A is closed if and only if $A \supseteq D(A)$.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. State and prove the Cauchy criterion for Riemann integrability.

22. Prove that if (f_n) is a sequence of functions in $\mathcal{R}[a, b]$ and (f_n) converges uniformly on [a,b] to f, then $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n\to\infty} \int_a^b f_n$.

23. Show that in a complete metric space X, if $\{F_n\}$ is a decreasing sequence of non-empty closed subsets of X such that $d(F_n) \to 0$, then $\cap_{n=1}^{\infty} F_n$ contains exactly one point. Give an example to show that the condition $d(F_n) \to 0$ can not be dropped to obtain the result.

24. Show that a subset of a topological space is dense if and only if it intersects every non-empty open set.

K19U 0122

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2019
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B10 MAT : Linear Algebra

Time : 3 Hours　　　　Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Give an example of a proper non trivial subspace of P(R), the vectorspace of all polynomials with real coefficients.

2. A subset of a linearly dependent set can possibly be linearly independent. Justify by giving an example.

3. The null space of the operator $T : R^2 \mapsto R^2$ given by $T(a_1, a_2) = (a_1, 0)$ is

4. The number of linearly independent solutions of the equation $x + y + z = 0$ is

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. In any vectorspace V show that $(a + b)(x + y) = ax + bx + ay + by$ for all scalars a and b and all vectors x and y.

6. Let $V = R^2 = R \times R$ where vector addition and scalar multiplication are defined by; $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$ and $r(x_1, x_2) = (rx_1, x_2)$.
Is V a vectorspace over R ? Justify.

7. Show that any intersection of subspaces of a vectorspace V is again a subspace of V.

8. Let $T : V \mapsto W$ be a linear transformation. Prove that N(T), the nullspace of T, is a subspace of V.

9. Let $T : V \mapsto W$ be an invertible linear transformation. Use dimension theorem to observe that $\dim V = \dim W$.

P.T.O.

10. Let V = C[a,b] be the vectorspace of all continuous real valued functions defined over the closed bounded interval [a, b]. Describe the fundamental theorem of calculus interms of linear transformations on V.

11. Find the values of λ for which the following system of equations have non zero solutions.
$\lambda x + 8y = 0$
$2x + \lambda y = 0$

12. Verify that the set of all characteristic vectors of a square matrix associated with a fixed characteristic value λ is a subspace of the respective Eucledian space.

13. Use Gauss elimination to solve the system of equations :
$2x + y + z = 10$
$3x + 2y + 3z = 18$
$x + 4y + 9z = 16$

14. Use Gauss Jordan elimination to solve the system of equations :
$2x + y + z = 10$
$3x + 2y + 3z = 18$
$x + 4y + 9z = 16$

SECTION – C

Answer **any 4** questions from among the questions **15** to **20.** These questions carry **4** marks **each.**

15. Define vectorspace and show that in every vector space (–1) x is the additive inverse of x.

16. Define a basis of a vectorspace. Give an example of a basis of $M_{2\times 2}(R)$.

17. Let V and W be vectorspaces and let $T : V \mapsto W$ be linear. Then prove that T is one to one if and only if $N(T) = \{0\}$.

18. Suppose that AX = B has a solution. Show that this solution is unique if and only if AX = 0 has only the trivial solution.

19. Test the following system of equations for consistency and solve it if it is consistent.

$x + 2y + 3z = 14$
$3x + y + 2z = 11$
$2x + 3y + z = 11$

20. Find the largest characteristic value and a corresponding characteristic vector of the matrix.

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. If S is a nonempty subset of a vectorspace V, then show that span (S) is a subspace of V and is the smallest subspace of V containing S. Under what further condition S can become a basis of V ?

22. Let V and W be vectorspaces over a common field F and suppose that V has a basis $\{x_1, x_2, \ldots, x_n\}$. Prove that for any fixed vectors $y_1, y_2, \ldots, y_n$ in W there exists exactly one linear transformation $T : V \mapsto W$ such that $T(x_i) = y_i$ for $i = 1 \ldots n$.

23. Show that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

satisfies Cayley Hamilton theorem and hence obtain A^{-1}.

24. Prove that

$$A = \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

is diagonalizable and find the diagonal form.

K19U 0123

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.) Examination, April 2019
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B11MAT – Numerical Methods and Partial Differential Equations

Time : 3 Hours

Max. Marks: 48

SECTION – A

All the **4** questions are **compulsory**. They carry **1** mark **each**.

1. Give the condition for convergence in the General Iteration method.

2. Complete the expression $\nabla = \ldots - E^{-1}$.

3. State Simpson's 1/3 rule of integration.

4. Give the $(n + 1)^{th}$ approximation step in Picard's method.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Find an interval which contains the root of the equation $2x^3 - x^2 + x - 6 = 0$.

6. Find a root of the equation $\log x - \cos x = 0$, where x is in radians, correct to two decimal places, using Regula Falsi Method.

7. Using Lagrange's interpolation formula find the approximate value of $\sin(\pi/6.)$.

x	0	$\pi/4$	$\pi/2$
y = sin x	0	0.7071	1

P.T.O.

8. Show that $\Delta\left(\frac{f_i}{g_i}\right) = -\frac{g_i \Delta f_i - f_i \Delta g_i}{g_i g_{i+1}}$.

9. Find the approximate value of $\int_0^{\pi} \sin x dx$ using trapezoidal rule by dividing the range of integration into six equal parts.

10. The acceleration of a missile during its first 40 seconds of flight is given in the following table. Find the velocity of the missile when t = 40s.

t(s)	0	10	20	30	40
a(m/s^2)	30	31.63	33.34	35.47	37.75

11. Write the formula for Runge Kutta Method of order 2.

12. Given $y' = x - y^2$ and y(0) = 1, find y(0.1) using Taylor's series method, correct to two decimal places.

13. Give the Fourier series solution of the one dimensional heat equation, with both ends of the bar kept at temperature 0 and the initial temperature function along the bar is f(x).

14. Solve the equation $u_{yy} = 0$ where u is a function of x and y.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**.

15. Find a root of the equation 2x = cos x + 3, where x is in radians, correct to two decimal places, using General iteration method.

16. Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$.

17. From the following table find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 3.5.

x	2	2.5	3	3.5	4
y	12	20.125	32	48.375	70

18. Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$, with $y(0) = 0$, use Picard's method to find y when $x = 0.5$.

19. Use modified Euler's method to the equation $\frac{dy}{dt} = t + \sqrt{y}$, $y(0) = 1$ to find $y(0.2)$ using three iterations taking $h = 0.2$.

20. Determine the solution of the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, where

$u(0, t) = 0$, $u(l, t) = 0$ and $u(x, 0) = x$, l being the length of the bar.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each.**

21. a) Use Newton Raphson method to find $(-10)^{1/3}$, correct to two decimal places.

    b) Find a real root of $x^3 - 3x - 5 = 0$ using Bisection method.

22. Values of x (in degrees) and sin x are given in the following table :

x	15	20	25	30	35	40
y = sin x	0.2588	0.3420	0.4226	0.5	0.5736	0.6428

Determine the value of sin 38° using Newton's backward difference interpolation formula.

23. Use Fourth order Runge-Kutta method to the equation $\frac{dy}{dt} = t + y$, with $y(0) = 1$ to find $y(0.1)$ and $y(0.2)$.

24. Find the solution u(x, t) of the wave equation with initial deflection

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \\ \frac{2k}{L}(L - x) & \text{if } \frac{L}{2} < x < \end{cases}$$

and initial velocity 0.

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Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.)
Examination, April 2020
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B12MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Sketch the region $\{z : \text{Re}(iz) \geq 0\}$.

2. Define Harmonic function.

3. Find the Radius of convergence of $\sum 7^n z^n$.

4. Find the residue of $f(z) = e^z$ at $z = 0$.

SECTION – B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Give an example of a function which is differentiable exactly at one point and give its justification.

6. Verify Cauchy-Riemann equations for the function $f(z) = z^2$.

7. Evaluate $\int_C |z|\, dz$, where C is the line segment from origin to 1 + i.

8. Find the Radius of convergence of $\sum (1 + i)^n (z - 3i)^n$.

9. Find the residue of $f(z) = \dfrac{9z + i}{z(z^2 + 1)}$ at $z = i$.

P.T.O.

10. Find the Laurent's series expansion of $f(z) = \frac{1}{z^5} \sin z$ with center 0.

11. State Taylors Theorem. Find the Taylors series expansion of $f(z) = \frac{1}{1+z^2}$ centered at $z = 0$.

12. Give an example of a series which is convergent but not absolutely. Give justification.

13. State Laplace's Equation. Give an example of a real valued function which satisfy Laplace's Equation on the complex plane.

14. State Cauchy's inequality.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each.**

15. Prove that an analytic function of constant absolute value is constant in a domain.

16. Evaluate the following :

   a) $\int_{0}^{1-i} z^2 dz$

   b) $\int_{8+\pi i}^{8-3\pi i} e^{z/2} dz$

17. The power series $\Sigma a_n z^n$ converge at $z = 1$ and diverge at $z = -1$. Find the radius of convergence of $\Sigma a_n z^n$.

18. State and prove Residue Theorem.

19. Find an analytic function $f(z) = u(x, y) + iv(x, y)$, where $u(x, y) = xy$.

20. State and prove the theorem of convergence of power series.

SECTION – D

Answer **any 2** questions from among the questions **21 to 24**. These questions carry **6** marks **each**.

21. State and prove Cauchy – Riemann equations.

22. a) Define singular point, isolated singular point, removable singular point, pole and essential singular point.

    b) Give an example of a non-isolated singular point.

23. a) State and prove Cauchy's integral formula.

    b) Evaluate $\int_C \frac{e^z}{z-2}\,dz$, where C is the circle $|z| = 3$.

24. Give examples and justifications of power series having Radius of convergence 1 and

    a) Which diverge at every point on the circle of convergence ?

    b) Which doesn't diverge at every point on the circle of convergence ?

K20U 0130

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2020
(2014 Admission Onwards)
Core Course in Mathematics
6B13MAT : MATHEMATICAL ANALYSIS AND TOPOLOGY

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. If $I = [0, 4]$, calculate the norm of the partition $P = \{0, 1, 1.5, 2, 3.4, 4\}$.

2. Evaluate $\lim (f_n(x))$ where $f_n(x) = \frac{x}{x-n}$ for all $x \geq 0$, $n \in \mathbb{N}$.

3. Fill in the blanks : The closure of set of all irrational numbers is ________

4. Write a pair of topologies T_1 and T_2 on $X = \{a, b, c\}$ so that $T_1 \cup T_2$ is not a topology on X.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each.**

5. Show that every constant real valued function on [a, b] is in $\mathcal{R}$ [a, b].

6. State squeeze theorem for Riemann integrability.

7. Find the value of $\int_{-1}^{10} \text{sgn}(x)\, dx$.

8. Prove that the sequence of functions, $f_n(x) = \frac{x}{n}$, $n \in \mathbb{N}$ converges uniformly on [0, 1]

9. State the Bounded Convergence Theorem.

10. Define a metric space and write an example.

11. Prove that in a metric space each open sphere is an open set.

12. Give an example of a pair of subsets A and B of the real line with usual topology such that $\text{Int}(A) \cup \text{Int}(B) \neq \text{Int}(A \cup B)$.

13. Define subspace of a topological space and show that it is a topological space.

14. Is the real line $\mathbb{R}$ with the usual topology separable ? Justify.

P.T.O.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**.

15. Prove that if $f : [a, b] \to \mathbb{R}$ is continuous on [a, b], then $f \in \mathcal{R}[a, b]$.

16. State and prove composition theorem in Riemann integrals. Deduce that if $f \in \mathcal{R}[a, b]$, then $|f| \in \mathcal{R}[a, b]$ and $\left|\int_a^b f\right| \leq \int_a^b |f|$.

17. Prove that a power series $\sum a_n x^n$ is absolutely convergent if $|x| < R$ and is divergent if $|x| > R$. (Here R is the radius of convergence and assume that $0 < R < \infty$).

18. Show that a subset F of a metric space is closed if and only if its complement F' is open.

19. Prove that a subspace Y of a complete metric space X is complete if and only if it is closed.

20. Prove that in a topological space $\overline{A} = A \cup D(A)$ and A is closed if and only if $A \supseteq D(A)$.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. State and prove the Cauchy criterion for Riemann integrability.

22. Prove that if (f_n) is a sequence of functions in $\mathcal{R}[a, b]$ and (f_n) converges uniformly on [a,b] to f, then $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n \to \infty} \int_a^b f_n$.

23. Show that in a complete metric space X, if $\{F_n\}$ is a decreasing sequence of non-empty closed subsets of X such that $d(F_n) \to 0$, then $\cap_{n=1}^{\infty} F_n$ contains exactly one point. Give an example to show that the condition $d(F_n) \to 0$ can not be dropped to obtain the result.

24. Show that a subset of a topological space is dense if and only if it intersects every non-empty open set.

K19U 0122

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VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2019
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B10 MAT : Linear Algebra

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Give an example of a proper non trivial subspace of P(R), the vectorspace of all polynomials with real coefficients.

2. A subset of a linearly dependent set can possibly be linearly independent. Justify by giving an example.

3. The null space of the operator $T : R^2 \mapsto R^2$ given by $T(a_1, a_2) = (a_1, 0)$ is

4. The number of linearly independent solutions of the equation $x + y + z = 0$ is

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. In any vectorspace V show that $(a + b)(x + y) = ax + bx + ay + by$ for all scalars a and b and all vectors x and y.

6. Let $V = R^2 = R \times R$ where vector addition and scalar multiplication are defined by; $(x_1, x_2) + (y_1, y_2) = (x_1 + y_1, x_2 + y_2)$ and $r(x_1, x_2) = (rx_1, x_2)$.
Is V a vectorspace over R ? Justify.

7. Show that any intersection of subspaces of a vectorspace V is again a subspace of V.

8. Let $T : V \mapsto W$ be a linear transformation. Prove that N(T), the nullspace of T, is a subspace of V.

9. Let $T : V \mapsto W$ be an invertible linear transformation. Use dimension theorem to observe that dimV = dimW.

P.T.O.

10. Let V = C[a,b] be the vectorspace of all continuous real valued functions defined over the closed bounded interval [a, b]. Describe the fundamental theorem of calculus interms of linear transformations on V.

11. Find the values of λ for which the following system of equations have non zero solutions.
$\lambda x + 8y = 0$
$2x + \lambda y = 0$

12. Verify that the set of all characteristic vectors of a square matrix associated with a fixed characteristic value λ is a subspace of the respective Eucledian space.

13. Use Gauss elimination to solve the system of equations :
$2x + y + z = 10$
$3x + 2y + 3z = 18$
$x + 4y + 9z = 16$

14. Use Gauss Jordan elimination to solve the system of equations :
$2x + y + z = 10$
$3x + 2y + 3z = 18$
$x + 4y + 9z = 16$

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Define vectorspace and show that in every vector space (–1) x is the additive inverse of x.

16. Define a basis of a vectorspace. Give an example of a basis of $M_{2\times2}(R)$.

17. Let V and W be vectorspaces and let $T : V \mapsto W$ be linear. Then prove that T is one to one if and only if $N(T) = \{0\}$.

18. Suppose that AX = B has a solution. Show that this solution is unique if and only if AX = 0 has only the trivial solution.

19. Test the following system of equations for consistency and solve it if it is consistent.

$x + 2y + 3z = 14$
$3x + y + 2z = 11$
$2x + 3y + z = 11$

20. Find the largest characteristic value and a corresponding characteristic vector of the matrix.

$$\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. If S is a nonempty subset of a vectorspace V, then show that span (S) is a subspace of V and is the smallest subspace of V containing S. Under what further condition S can become a basis of V ?

22. Let V and W be vectorspaces over a common field F and suppose that V has a basis $\{x_1, x_2, \ldots, x_n\}$. Prove that for any fixed vectors $y_1, y_2, \ldots, y_n$ in W there exists exactly one linear transformation $T : V \mapsto W$ such that $T(x_i) = y_i$ for $i = 1 \ldots n$.

23. Show that the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$$

satisfies Cayley Hamilton theorem and hence obtain A^{-1}.

24. Prove that

$$A = \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

is diagonalizable and find the diagonal form.

K19U 0123

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VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.) Examination, April 2019
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B11MAT – Numerical Methods and Partial Differential Equations

Time : 3 Hours

Max. Marks: 48

SECTION – A

All the **4** questions are **compulsory**. They carry **1** mark **each**.

1. Give the condition for convergence in the General Iteration method.
2. Complete the expression $\nabla = ... - E^{-1}$.
3. State Simpson's 1/3 rule of integration.
4. Give the $(n + 1)^{th}$ approximation step in Picard's method.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Find an interval which contains the root of the equation $2x^3 - x^2 + x - 6 = 0$.
6. Find a root of the equation $\log x - \cos x = 0$, where x is in radians, correct to two decimal places, using Regula Falsi Method.
7. Using Lagrange's interpolation formula find the approximate value of $\sin(\pi/6.)$.

x	0	$\pi/4$	$\pi/2$
y = sin x	0	0.7071	1

P.T.O.

8. Show that $\Delta\left(\frac{f_i}{g_i}\right) = -\frac{g_i \Delta f_i - f_i \Delta g_i}{g_i g_{i+1}}$.

9. Find the approximate value of $\int_0^{\pi} \sin x dx$ using trapezoidal rule by dividing the range of integration into six equal parts.

10. The acceleration of a missile during its first 40 seconds of flight is given in the following table. Find the velocity of the missile when t = 40s.

t(s)	0	10	20	30	40
a(m/s²)	30	31.63	33.34	35.47	37.75

11. Write the formula for Runge Kutta Method of order 2.

12. Given $y' = x - y^2$ and $y(0) = 1$, find $y(0.1)$ using Taylor's series method, correct to two decimal places.

13. Give the Fourier series solution of the one dimensional heat equation, with both ends of the bar kept at temperature 0 and the initial temperature function along the bar is f(x).

14. Solve the equation $u_{yy} = 0$ where u is a function of x and y.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**.

15. Find a root of the equation 2x = cos x + 3, where x is in radians, correct to two decimal places, using General iteration method.

16. Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$.

17. From the following table find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 3.5.

x	2	2.5	3	3.5	4
y	12	20.125	32	48.375	70

18. Given the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$, with y(0) = 0, use Picard's method to find y when x = 0.5.

19. Use modified Euler's method to the equation $\frac{dy}{dt} = t + \sqrt{y}$, y(0) = 1 to find y(0.2) using three iterations taking h = 0.2.

20. Determine the solution of the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, where

u(0, t) = 0, u(*l*, t) = 0 and u(x, 0) = x, *l* being the length of the bar.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each.**

21. a) Use Newton Raphson method to find $(-10)^{1/3}$, correct to two decimal places.

    b) Find a real root of $x^3 - 3x - 5 = 0$ using Bisection method.

22. Values of x (in degrees) and sin x are given in the following table :

x	15	20	25	30	35	40
y = sin x	0.2588	0.3420	0.4226	0.5	0.5736	0.6428

Determine the value of sin 38° using Newton's backward difference interpolation formula.

23. Use Fourth order Runge-Kutta method to the equation $\frac{dy}{dt} = t + y$, with y(0) = 1 to find y(0.1) and y(0.2).

24. Find the solution u(x, t) of the wave equation with initial deflection

$$f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \\ \frac{2k}{L}(L-x) & \text{if } \frac{L}{2} < x < \end{cases}$$

and initial velocity 0.

K19U 0124

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VI Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, April 2019
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B12 MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Write the polar form of the complex number $z = 1 + i$, using principle value of the argument.
2. Write the triangle inequality of complex numbers.
3. Find the Radius of convergence of $\sum n z^n$.
4. Give an example of a function having a simple pole at origin.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Verify Cauchy-Riemann equations for the function $f(z) = z^3$.
6. Does there exist a function in the complex plane which is analytic exactly at one point ? Give justification.
7. Evaluate $\int_C e^z \, dz$, where C is the line segment from origin to $1 + i$.
8. Evaluate $\int_C \frac{1}{z-i} dz$, using Cauchy's integral formula, where C is the circle $|z| = 2$.

P.T.O.

9. Find the radius of convergence of $\sum \frac{(2n)!}{(n!)^2}(z-3i)^n$.

10. Find the Laurent's series expansion of $f(z)=\frac{1}{z^3-z^4}$ about z = 0 in the region $0 < |z| < 1$.

11. Find the residue of f(z) = cotz at z = 0.

12. State Taylors Theorem. Find the Taylors series expansion of $f(z) = e^z$ centered at z = 0.

13. Define Essential singularity. Give one example of a function having essential singularity at z = 0.

14. Give an example of a series which is convergent but not absolutely. Give justification.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Prove that an analytic function whose modulus constant is constant in a domain.

16. State Cauchy's Integral Formula . Using this evaluate $\int_C \frac{z^3-6}{2z-i}dz$,

where $C := |z| = 1$.

17. State and prove Morera's Theorem.

18. State Cauchy-Hadamard formula for Radius of convergence. Using this Evaluate the radius of convergence of $\sum\left(\frac{a}{b}\right)^n(z-3i)^n$.

19. a) State Laurent's Theorem.

b) Find the Residue of $f(z) = z^2 e^{\frac{1}{z}}$ with center 0.

20. a) State comparison test for convergence of a series.

b) Discuss the convergence of the series $\sum \frac{\sin n}{3^n} z^n$.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. a) Define Analytic function.

b) Give an example of a function which satisfy Cauchy-Riemann equation at origin but not analytic at origin and justification.

22. State and prove Cauchy's Integral formula.

23. Give examples and justifications of power serieses having Radius of convergence 1 and

a) which diverge at every point on the circle of convergence

b) which doesn't diverge at every point on the circle of convergence .

24. State and prove Residue theorem.

K19U 0125

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VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.) Examination, April 2019
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B13MAT : Mathematical Analysis and Topology

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Define the Riemann sum of a function $f : [a, b] \to \mathbb{R}$ corresponding to a tagged partition $\dot{P} = \{([x_i - 1, x_i], t_i)\}_{i=1}^{n}$.

2. Find the radius of convergence of $\sum \frac{x^n}{n}$.

3. State True or False: The subspace (0, 1] of $\mathbb{R}$ with usual metric is a complete metric space.

4. Suppose that T is the discrete topology on X = {a, b, c, d} and A = {b, c}. Then find Int(A).

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. If $f \in R[a, b]$ and $|f(x)| \leq M$ for all $x \in [a, b]$, then show that $\left|\int_a^b f\right| \leq M(b - a)$.

6. Show that Thomae's function, $f : [0, 1] \to \mathbb{R}$ given below is Riemann integrable over [0, 1].

$$f(x) = \begin{cases} 0, \text{ when } x \text{ is irrational} \\ 1, \text{ when } x = 0 \\ \frac{1}{n}, \text{ when } x = \frac{m}{n} \text{ is rational and is in the lowest form} \end{cases}.$$

P.T.O.

7. Prove that if f and g belong to R[a, b], then the product fg belongs to R[a, b].

8. Test the uniform convergence of the sequence of functions, $f_n(x) = \frac{x}{n}$, $n \in \mathbb{N}$ on [0, 1].

9. Prove that if a sequence of continuous functions (f_n) defined on $A \subseteq \mathbb{R}$ converges uniformly on A to a function f, then f is continuous on A.

10. Show that in a metric space each open sphere is an open set.

11. Describe the Cantor set and show that it is closed in $\mathbb{R}$.

12. Prove that if a convergent sequence in a metric space has infinitely many distinct points, then its limit is a limit point of the set of terms of the sequence.

13. Prove that in the class of all topological spaces the relation, ~ defined by X ~ Y iff X and Y are homeomorphic is an equivalence relation.

14. Is the union of two topologies on a set a topology ? Justify.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Show that if $f : [a, b] \to \mathbb{R}$ is monotone on [a, b], then $f \in R$ [a, b].

16. Using the substitution theorem evaluate $\int_1^4 \frac{\sin\sqrt{t}}{\sqrt{t}}\,dt$.

17. State and prove Cauchy criterion for uniform convergence.

18. Show that in a metric space X any finite intersection of open subsets of X is open in X. Give an example to show that in a metric space, a countable intersection of open sets need not be open.

19. Define the closure of a set in a metric space, give an example and show that closure of a set A is the smallest closed set containing A.

20. Let $f : X \to Y$ be a mapping of one topological space into another. Show that f is continuous if and only if f^{-1} (F) is closed in X whenever F is closed in Y.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Prove that if f, g : [a, b] → $\mathbb{R}$ are Riemann integrable on [a, b], then f + g is also integrable on [a, b].

22. If f_n : [a, b] → $\mathbb{R}$ are Riemann integrable over [a, b] for every n ∈ $\mathbb{N}$ and Σf_n converges to f uniformly on [a, b], then show that f is Riemann integrable and $\int_a^b f = \sum_{n=1}^{\infty} \int_a^b f_n$.

23. If $\{A_n\}$ is a sequence of nowhere dense subsets in a complete metric space X, then prove that there exists a point in X which is not in any of the A_n's.

24. Let X be a non-empty set and C be a class of subsets of X which is closed under the formation of arbitrary intersections and finite unions. Prove that there exists a topology on X such that the class of all closed subsets of the space X coincides with C.

K19U 0126

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VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.) Examination, April 2019
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B14 MAT : (Elective – A) : Operations Research

Time : 3 Hours   Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Define positive semi definite quadratic form.
2. Define the term feasible solution of a linear programming problem.
3. What is an unbalanced transportation problem ?
4. Define two person zero sum game.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Show that $S = \{(x_1, x_2) : x_1^2 + x_2^2 \leq 4\}$ is a convex set.
6. Write the quadratic form $x_1^2 + 2x_2^2 - 7x_3^2 - 4x_1x_2 + 3x_1x_3 - 5x_2x_3$ in the form X^TAX.
7. Obtain all basic solutions to the following system of linear equations :

   $x_1 + 2x_2 + x_3 = 4 : 2x_1 + x_2 + 5x_3 = 5.$
8. State the general LPP in the standard form.

P.T.O.

9. Give a mathematical formulation of the transportation problem.

10. Explain loops in transportation tables.

11. Explain the difference between transportation problem and assignment problem.

12. What is no passing rule in a sequencing algorithm ?

13. What are the properties of a game ?

14. Explain the concept of value of the game.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each.**

15. Prove that the set of all convex combinations of a finite number of points $S \subset R^n$ is a convex set.

16. A firm manufactures two types of products A and B and sells them at a profit of Rs. 2 on type A and Rs. 3 on type B. Each product is processed on two machines G and H. Type A requires 1 minute of processing time on G and 2 minutes on H; type B requires 1 minute on G and 1 minute on H. The machine G is available for not more than 6 hours 40 minutes while machine H is available for 10 hours during any working day. Formulate the problem as a linear programming problem.

17. What are the methods for finding initial basic feasible solution of the transportation problem ? Explain any one.

18. Describe a method of drawing minimum number of lines in the context of assignment problem.

19. What are the main assumptions made while dealing with sequencing problem ?

20. Find the saddle point of the payoff matrix.

$$\begin{pmatrix} 4 & 1 & -3 \\ 3 & 2 & 5 \\ 0 & 1 & 6 \end{pmatrix}$$

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Solve using simplex method :

    Maximize $Z = 5x_1 + 3x_2$

    Subject to $3x_1 + 5x_2 \leq 15$

    $5x_1 + 2x_2 \leq 10$ and $x_1 \geq 0, x_2 \geq 0$.

22. Describe MODI method in transportation problem.

23. Solve the following assignment problem ?

	1	**2**	**3**	**4**
A	49	60	45	61
B	55	63	45	61
C	52	62	49	68
D	55	64	48	66

24. Solve the following 2×3 game graphically.

Player B

Player A $\begin{pmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{pmatrix}$

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K20U 0131

VI Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.)
Examination, April 2020
(2014 Admission Onwards)
CORE COURSE IN MATHEMATICS
6B14MAT (Elective A) : Operations Research

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Define global minimum of a function f(x).

2. What do you mean by degeneracy in a linear programming problem ?

3. What is assignment problem ?

4. Define saddle point of a game.

SECTION – B

Answer any 8 questions from among the questions 5 to 14. These questions carry 2 marks each.

5. Show that the function $f((x_1, x_2)) = x_1^2 + x_2^2$ is a convex function over all of R^2.

6. Determine whether the quadratic form $2x_1^2 + 6x_2^2 - 6x_1x_2$ is positive definite or negative definite.

7. Define the term basic solution. How many basic solutions are there to a given system of two simultaneous linear equation in four unknowns ?

8. State the general LPP in the canonical form.

9. Explain least cost method to solve transportation problem for an initial solution.

P.T.O.

10. What is degeneracy in transportation problems ?

11. Give two applications of assignment problem.

12. Define the sequencing problem with n jobs and two machines.

13. What assumptions are made in the theory of games ?

14. Explain the dominance property in game theory.

SECTION – C

Answer any 4 questions from among the questions 15 to 20. These questions carry 4 marks each.

15. Let f(x) be a convex function on a convex set S. Prove that f(x) has a local minimum on S, then this local minimum is also a global minimum on S.

16. Solve graphically Max $Z = 80x_1 + 55x_2$

    Subject to $4x_1 + 2x_2 \leq 40$

    $2x_1 + 4x_2 \leq 32 \quad x_1 \geq 0, x_2 \geq 0.$

17. Obtain an initial basic feasible solution to the following transportation problem :

	D	E	F	G	available
A	11	13	17	14	250
B	16	18	14	10	300
C	21	24	13	10	400
requirement	200	225	275	250	

18. Show that the optimal solution of a assignment problem is unchanged if we add or subtract the same constant to the entries of any row or column of the cost matrix.

19. Explain the sequencing problem with n jobs and k machines.

20. Explain the graphical method of solving a game.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each.**

21. Define the dual of a linear programming problem. Prove that the dual of the dual is the primal.

22. Solve the following transportation problem.

	X	Y	Z	Availability
A	50	30	220	1
B	90	45	170	3
C	250	200	50	4
requirement	4	2	2	

23. Explain the Hungarian method to solve an assignment problem.

24. Describe the procedure to solve any 2×2 two person zero sum game without any saddle point.