K19U 2254

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.)
Examination, November - 2019
(2014 Admn. Onwards)
Core Course in Mathematics
5B05 MAT : REAL ANALYSIS

Time : 3 Hours Max. Marks : 48

SECTION - A

Instructions: Answer **all** questions. Each question carries **One** mark. **(4×1=4)**

1. State Arithmetic-Geometric Mean Inequality.
2. Find $\sup\left\{\frac{1}{m}-\frac{1}{n} : n \in \mathbb{N}\right\}$.
3. Show that the sequence $\left(\frac{1}{n}\right)$ is a Cauchy sequence.
4. State Weierstrass Approximation theorem.

SECTION - B

Answer any **Eight** questions. Each carries **Two** marks. **(8×2=16)**

5. State and prove triangle inequality.
6. Show that the sequence (2^n) does not converges.
7. Show that the series $\sum_{n=1}^{\infty} \cos n$ is divergent.
8. Show that if a convergent series contains only a finite number of negative terms, then prove that it is absolutely convergent.
9. Let $X = (x_n)$ be a nonzero sequence in $\mathbb{R}$ and let $a = \lim\left(n\left(1-\left|\frac{x_{n-1}}{x_n}\right|\right)\right)$, whenever the limit exists. Then prove that $\sum x_n$ is absolutely convergent when $a > 1$ and is not absolutely convergent when $a < 1$.

P.T.O.

10. What do you mean by saying that a function is continuous at a point c.

11. Let $A \subset \mathbb{R}$, let f and g be functions on A to $\mathbb{R}$, and if $g(x) \neq 0$ for all $x \in \mathbb{R}$. Suppose that $c \in A$ and that f and g are continuous at c. Then show that f/g is continuous at c.

12. Give an example of two functions f and g that are both discontinuous at a point c in $\mathbb{R}$ such that the sum $f + g$ is continuous at c.

13. If $f : A \to \mathbb{R}$ is a Lipschitz function, then prove that f is uniformly continuous on A.

14. Let $I \subset \mathbb{R}$ be an interval and $f : I \to \mathbb{R}$ be increasing on I. If $c \in I$ then prove that f is continuous at c if and only if $j_f(c) = 0$, where $j_f(c)$ is the jump of f at c.

SECTION - C

Answer any **Four** questions. Each carries **Four** marks. **(4×4=16)**

15. Prove that the set $\mathbb{R}$ of real numbers is not countable.

16. Let $X = (x_n)$ and $Z = (z_n)$ be sequences of real numbers that converges to x and z, respectively, where z_n and z are nonzero real numbers. Then show that X/Z converges to x/z.

17. Let A be an infinite subset of $\mathbb{R}$ that is bounded above and let $u = \sup A$. Show that there exists an increasing sequence (x_n) with $x_n \in A$ for all $n \in \mathbb{N}$ such that $u = \lim(x_n)$.

18. Show that every contractive sequence is a Cauchy sequence and therefore is convergent.

19. State and prove Abel's test for the convergence of the product of two series.

20. Let $I = [a, b]$ be a closed, bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. If $k \in \mathbb{R}$ is any number satisfying $\inf f(I) \leq k \leq \sup f(I)$, then prove that there exists a number $c \in I$ such that $f(c) = k$.

SECTION - D

Answer any **Two** questions Each question carries **six** marks.**(2×6=12)**

21. a) If $S = \left\{ \frac{1}{n} : n \in S \right\}$ then prove that inf $S = 0$. **(2)**

b) Prove that there exists a positive number x such that $x^2 = 2$. **(4)**

22. a) Let $X = (x_n : n \in \mathbb{N})$ be a sequence of real numbers and let $m \in \mathbb{N}$. Then prove that the m-tail converges if X converges. **(2)**

b) Let $a > 0$. Construct a sequence s_n of real numbers that converges to $\sqrt{a}$. **(4)**

23. a) Let $X = (x_n)$ be a sequence in $\mathbb{R}$ and suppose that the limit $r = \lim |x_n|^{\frac{1}{n}}$ exists in $\mathbb{R}$. Then prove that Σx_n is absolutely convergent when $r < 1$ and is divergent when r > 1. **(3)**

b) Show that the absolute value function $f(x) = |x|$ is continuous at every point $c \in \mathbb{R}$. **(3)**

24. a) What do you mean by saying that a function is bounded on a subset of $\mathbb{R}$. Give an example of a bounded set. **(2)**

b) Show that every polynomial of odd degree with real coefficients has at leat one real root. **(4)**

K21U 1532

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Sup./Imp.) Examination, November 2021
(2015 – '18 Admns.)
CORE COURSE IN MATHEMATICS
5B05MAT : Real Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** carries **1** mark.

1. Write the Supremum of $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$.
2. Define contractive sequences.
3. Check the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.
4. State sequential criterion for continuity.

SECTION – B

Answer **any eight** questions. **Each** carries **2** marks.

5. Find all $x \in \mathbb{R}$ such that $\frac{2x+1}{x+2} < 1$.
6. If $x > -1$, show that $(1 + x)^n \geq 1 + nx\ \forall n \in \mathbb{N}$.
7. If $t > 0$, prove that there is an n_t in $\mathbb{N}$ such that $0 < \frac{1}{n_t} < t$.
8. Show that convergent sequences in $\mathbb{R}$ are bounded.
9. Suppose $X = (x_n)$, $Y = (y_n)$ and $Z = (z_n)$ are sequences in $\mathbb{R}$ such that $x_n \leq y_n \leq z_n \forall n \in \mathbb{N}$ and $\lim(x_n) = \lim(z_n)$. Show that $Y = (y_n)$ is convergent and $\lim(x_n) = \lim(y_n) = \lim(z_n)$.
10. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ converges.
11. Check the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

P.T.O.

12. State and prove Abel's Lemma.
13. Let I be an interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that f(I) is an interval.
14. If $f : I \to \mathbb{R}$ is uniformly continuous on a subset A of $\mathbb{R}$ and if (x_n) is a Cauchy sequence in A. Show that $(f(x_n))$ is also a Cauchy sequence in $\mathbb{R}$.

SECTION – C

Answer **any four** questions. **Each** carries **4** marks.

15. Show that the set $\mathbb{Q}$ of rational numbers is dense in the set $\mathbb{R}$ of real numbers.
16. For a, b $\in \mathbb{R}$, show that $|a + b| \leq |a| + |b|$ and deduce $\big||a| - |b|\big| \leq |a - b|$.
17. Let (x_n) be a sequence of real numbers such that $L = \lim \left(\frac{x_{n+1}}{x_n}\right)$ exists and let $L < 1$. Show that (x_n) converges and $\lim(x_n) = 0$.
18. State and prove the limit comparison test for series.
19. Let $Z = (z_n)$ be a decreasing sequence of strictly positive numbers with $\lim(z_n) = 0$. Show that the alternating series $\sum(-1)^{n+1}z_n$ is convergent.
20. Let $I = [a, b]$ be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that f has an absolute maximum and an absolute minimum on I.

SECTION – D

Answer **any two** questions. **Each** carries **6** marks.

21. a) State and prove nested intervals property.
    b) Using nested intervals property, show that [0, 1] is uncountable.
22. a) A sequence of real numbers is convergent if and only if it is a Cauchy sequence. Prove.
    b) Show that $\lim\left(n^{1/n}\right) = 1$.
23. a) State and prove Raabe's test.
    b) If a and b are positive numbers, show that $\sum(an + b)^{-p}$ converges if $p > 1$ and diverges if $p \leq 1$.
24. a) Let $I = [a, b]$ and let $f : I \to \mathbb{R}$ be continuous on I. If $f(a) < 0 < f(b)$, then there exists a number $c \in (a, b)$ such that $f(c) = 0$.
    b) Show that every polynomial of odd degree with real coefficients has at least one real root.

K20U 1532

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2020 (2014 Admn. Onwards) CORE COURSE IN MATHEMATICS 5B05MAT : Real Analysis

Time : 3 Hours

Max. Marks : 48

SECTION – A

(Answer **all** the questions. **Each** carries **1** mark)

1. Find all real x so that $|x - 1| < |x|$.

2. Give two divergent sequences (x_n) and (y_n) such that $(x_n + y_n)$ is convergent.

3. State n^{th} term test.

4. Show that $f(x) = \frac{1}{x}, \forall x$ is not uniformly continuous on $(0, \infty)$. **(4×1=4)**

SECTION – B

(Answer **any eight** questions. **Each** carries **2** marks)

5. There does not exists a rational number r such that $r^2 = 2$. Prove.

6. For positive real numbers a and b, show that $\sqrt{ab} \leq \frac{1}{2}(a + b)$, where equality occurring if and only if a = b.

7. Define infimum of a set. Find inf S if $S = \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$.

8. A sequence in $\mathbb{R}$ can have atmost one limit. Prove.

P.T.O.

9. Prove that every Cauchy sequence is bounded.

10. If Σx_n and Σy_n are convergent, show that the series $\Sigma(x_n + y_n)$ is convergent.

11. Check the convergence of $\sum_{n=1}^{\infty} \frac{1}{n^p}$.

12. If $X = (x_n)$ is a decreasing sequence of real numbers with $\lim x_n = 0$, and if the partial sums (s_n) of Σy_n are bounded, prove that the series $\Sigma x_n y_n$ is convergent.

13. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that the set $f(I) = \{f(x) : x \in I\}$ is a closed bounded interval.

14. Give an example to show that every uniformly continuous functions are not Lipschitz functions. **(8×2=16)**

SECTION – C

(Answer **any four** questions. **Each** carries **4** marks)

15. State and prove Archimedean property of $\mathbb{R}$.

16. If S is a subset of $\mathbb{R}$ that contains at least two points and has the property

    If $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$.

    Show that S is an interval.

17. For $C > 0$, show that $\lim\left(C^{1/n}\right) = 1$.

18. Discuss the convergence of the Geometric series $\sum_{n=0}^{\infty} r^n$ for $r \in \mathbb{R}$.

19. If Σx_n is an absolutely convergent series in $\mathbb{R}$, show that any rearrangement Σy_k of Σx_n converges to the same value.

20. Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that f is uniformly continuous on I. **(4×4=16)**

SECTION – D

(Answer **any two** questions. **Each** carries **6** marks)

21. a) Prove the existence of a real number x such that $x^2 = 2$.

    b) If $a, b \in \mathbb{R}$, show that $||a| - |b|| \leq |a - b|$.

22. a) State and prove Bolzano Weierstrass Theorem for sequences.

    b) If $X = (x_n)$ is a bounded increasing sequence in $\mathbb{R}$, show that it converges and $\lim(x_n) = \sup\{x_n : n \in \mathbb{N}\}$.

23. a) State and prove D'Alembert ratio test.

    b) Check the convergence of the series whose n^{th} term is $\frac{(n!)^2}{(2n)!}$.

24. a) Let I be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. If $\varepsilon > 0$, then there exists step functions $s_\varepsilon : I \to \mathbb{R}$ such that $|f(x) - s_\varepsilon(x)| < \varepsilon, \forall x \in I$.

    b) Let $f(x) = x$, $\forall x \in [0, 1]$. Calculate the first few Bernstein polynomials for f.

(2×6=12)

K22U 2321

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2022
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
5B06MAT : Real Analysis – I

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. They carry **1** mark **each**.

1. Determine the set A of all real numbers x such that $2x + 3 \leq 6$.
2. Let $S = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$. Find inf S and sup S.
3. State monotone convergence theorem.
4. State alternating series test.
5. Prove that signum function *sgn* is not continuous at 0.

PART – B

Answer **any 8** questions from among the questions **6** to **16**. These questions carry **2** marks **each**.

6. Find all $x \in \mathbb{R}$ that satisfy $|x + 1| + |x - 2| = 7$.
7. State and prove triangle inequality.
8. If $x \in \mathbb{R}$, prove that there exists $n \in \mathbb{N}$ such that $x < n$.
9. State and prove squeeze theorem.
10. Let (x_n) be a sequence of positive real numbers such that $L = \lim \frac{x_{n+1}}{x_n}$ exists. If $L < 1$, prove that (x_n) converges and $\lim(x_n) = 0$.

P.T.O.

11. Prove that a Cauchy sequence of real numbers is bounded.

12. Prove that the sequence $\left(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}\right)$ is divergent.

13. Prove that $\sum_{n=0}^{\infty} r^n$ is convergent if $|r| < 1$ and divergent if $|r| \geq 1$.

14. Prove that $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+1}}$ is divergent.

15. Discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.

16. State and prove sequential criterion for continuity.

PART – C

Answer **any 4** questions from among the questions **17** to **23**. These questions carry **4** marks **each**.

17. Let S be a subset of $\mathbb{R}$ that contains atleast two points and has the property if $x, y \in S$ and $x < y$. Prove that $[x, y] \subseteq S$.

18. Let (x_n) and (y_n) be sequences of real numbers that converge to x and y respectively. Prove that $(x_n y_n)$ converges to xy.

19. Let $e_n = \left(1+\frac{1}{n}\right)^n$ for $n \in \mathbb{N}$. Prove that (e_n) is convergent.

20. Show that $\sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)} = \frac{1}{4}$.

21. State and prove Ratio test.

22. Prove that $g(x) = \sin\frac{1}{x}$ is continuous at every point $c \neq 0$.

23. State and prove boundedness theorem.

PART – D

Answer **any 2** questions from among the questions **24** to **27**. These questions carry **6** marks **each**.

24. a) State and prove nested interval property.

    b) Prove that $\mathbb{R}$ is not countable.

25. a) Prove that every contractive sequence is convergent.

    b) Let $f_1 = f_2 = 1$ and $f_{n+1} = f_n + f_{n-1}$. Define $x_n = \frac{f_n}{f_{n+1}}$. Prove that $\lim x_n = \frac{-1+\sqrt{5}}{2}$.

26. a) State and prove integral test.

    b) Let a and b be two positive numbers. Prove that $\Sigma(an + b)^{-p}$ converges if $p > 1$ and diverges if $p \leq 1$.

27. State and prove maximum minimum theorem.

K22U 1961

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Supplementary) Examination, November 2022
(2016-18 Admissions)
CORE COURSE IN MATHEMATICS
5B05 MAT : Real Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions, **each** question carries **one** mark.

1. State Supremum property of $\mathbb{R}$.
2. Prove that a sequence in $\mathbb{R}$ can have atmost one limit.
3. Prove that $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ converges.
4. Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \to \mathbb{R}$ be increasing on I. If $c \in I$, prove that f is continuous at c if and only if $j_f(c) = 0$.

SECTION – B

Answer **any eight** questions, **each** question carries **two** marks.

5. Determine the set $B = \{x \in \mathbb{R} : x^2 + x > 2\}$.
6. State and prove Bernoulli's inequality.
7. Let $S = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$. Find inf S and sup S.
8. Use the definition of the limit of a sequence to prove that $\lim\left(\frac{2n}{n+1}\right) = 2$.
9. State and prove squeeze theorem.

P.T.O.

10. Prove that $\sum_{n=0}^{\infty} r^n$ is convergent if $|r| < 1$ and divergent if $|r| \geq 1$.

11. Establish the convergence or divergence of the series whose n^{th} term is $\frac{n}{(n+1)(n+2)}$.

12. State and prove Dirichlet's test.

13. Prove that Dirichlet's function is discontinuous on $\mathbb{R}$.

14. State and prove Bolzano's intermediate value theorem.

SECTION – C

Answer **any four** questions, **each** question carries **four** marks.

15. State and prove Archimedean property.

16. State and prove nested interval property.

17. Let y_n be defined by $y_1 = 1, y_{n+1} = \frac{1}{4}(2y_n + 3)$ for $n \geq 1$. Prove that $\lim y_n = \frac{3}{2}$.

18. Prove that the p- series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges when $p > 1$.

19. State and prove integral test.

20. State and prove uniform continuity theorem.

SECTION – D

Answer **any two** questions, **each** question carries **six** marks.

21. Prove that there exists a positive real number x such that $x^2 = 2$.

22. Prove that every contractive sequence is a Cauchy sequence.

23. a) State and prove ratio test.

    b) Establish the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.

24. State and prove location of roots theorem.

K21U 4551

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021
(2019 Admn. Only)
CORE COURSE IN MATHEMATICS
5B06 MAT : Real Analysis – I

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **1** mark.

1. For $a, b \in \mathbb{R}$ if $a + b = 0$, then prove that $b = -a$.
2. Find the supremum of the set $\left\{1-\frac{1}{n} : n \in \mathbb{N}\right\}$.
3. Show that $\lim\limits_{n \to \infty} \frac{1}{n} = 0$.
4. Give an example of a discontinuous function on $\mathbb{R}$.
5. Define sequential criterion for the continuity of a function f on $\mathbb{R}$. **(4×1=4)**

PART – B

Answer **any eight** questions. **Each** question carries **2** marks.

6. State and prove Archimedean property.
7. Determine the set $B = \left\{x \in \mathbb{R} : x^2 + x > 2\right\}$.
8. Let $J_n = \left(0, \frac{1}{n}\right)$ for $n \in \mathbb{N}$, prove that $\bigcap_{n=1}^{\infty} J_n = \varnothing$.
9. Prove that a sequence in $\mathbb{R}$ can have at most one limit.
10. Show that a convergent sequence of real numbers is bounded.
11. Prove that every convergent sequence is a Cauchy sequence.

P.T.O.

12. If the series $\sum x_n$ converges, then prove that $\lim_{n \to \infty} x_n = 0$.

13. Check the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$.

14. Show that every absolutely convergent series is convergent.

15. Show that the function $f(x) = \frac{1}{x}$ is not bounded on the interval $(0, \infty)$.

16. If functions f, g are continuous at a point c, then prove that f + g is also continuous at c. **(8×2=16)**

PART – C

Answer **any four** questions. **Each** question carries **4** marks.

17. Prove that the set of real numbers is not countable.

18. State and prove Squeeze theorem.

19. State and prove Bolzano Weierstrass theorem.

20. Let $X = (x_n)$ and $Y = (y_n)$ that converges to x and y respectively, then prove that $X + Y$ converges to $x + y$.

21. Show that $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

22. If $X = (x_n)$ is a convergent monotone sequence and the series $\sum y_n$ is convergent, then prove that the series $\sum x_n y_n$ is convergent.

23. State and prove preservation of intervals theorem. **(4×4=16)**

PART – D

Answer **any two** questions. **Each** question carries **6** marks.

24. Prove that there exists a positive real number x such that $x^2 = 2$.

25. State and prove Monotone convergence theorem.

26. State and prove D'Alembert's ratio test for series.

27. If $f : [a, b] \to \mathbb{R}$ is a continuous function, then prove that f has an absolute maximum and absolute minimum on [a, b]. **(2×6=12)**