K19U 2257

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS- Reg./Sup./Imp.)

Examination, November-2019

(2014 Admn. Onwards)

Core Course in Mathematics

5B08 MAT: Vector Calculus

Time : 3 hrs Max. Marks : 48

SECTION - A

All the **4** questions are compulsory. They carry **1** mark each. **(4×1=4)**

1. Find the divergence of $e^x\left(\cos y\,\vec{i}+\sin y\,\vec{j}\right)$.

2. Express $\frac{\partial w}{\partial r}$ in terms of r and s if $w=x+y, x=r+s, y=r-s$.

3. What do you mean by a potential function for a vector field F.

4. Give a parametrization of the cone $z=\sqrt{x^2+y^2}, 0 \le z \le 1$.

SECTION - B

Answer any **8** questions among the questions 5 to 14. These questions carry **2** marks each. **(8×2=16)**

5. Find the angle between the planes $3x-6y-2z=15$ and $2x+y-2z=5$.

6. Show that $\vec{r}(t)=\cos t\,\vec{i}+\sqrt{5}\,\vec{j}+\sin t\,\vec{k}$ has constant length and is orthogonal to its derivative.

7. Define saddle point.

P.T.O.

8. Find the curl with respect to the right hand Cartesian coordinates of $yz\vec{i} + 3zx\vec{j} + z\vec{k}$.

9. Prove that for any twice continuously differentiable scalar function $f, curl\left(grad\ f\right) = \vec{0}$.

10. Find the local extreme values of the function $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$.

11. Show that $\vec{F} = (2x-3)\vec{i} - z\vec{j} + \cos z\vec{k}$ is not conservative.

12. Evaluate $f(x,y,z) = 3x^2 - 2y + z$ over the line segment C joining the origin to the point $(2,2,2)$.

13. Find the circulation of the field $F = (x-y)\vec{i} + x\vec{j}$ around the circle $\vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j}, 0 \le t \le 2\pi$.

14. Use Green's theorem to find the outward flux for the field $F = (x-y)\vec{i} + (y-x)\vec{j}$ across the curve square bounded by $x = 0, x = 1, y = 0, y = 1$.

SECTION - C

Answer any **4** questions among the questions **15 to 20.** These questions carry **4** marks each. **(4×4=16)**

15. Find and graph the osculating circle for a parabola $y = x^2$ at the origin.

16. Find the distance from $S(1,1,3)$ to the plane $3x + 2y + 6z = 6$.

17. Find the derivative of $f(x,y,z) = x^3 - xy^2 - z$ at $P_0(1,1,0)$ in the direction of $\vec{A} = 2\vec{i} - 3\vec{j} + 6\vec{k}$. Find the direction in which f increases most rapidly at P.

18. Use Taylor's formula to find a quadratic approximation of $f(x,y) = \cos x \cos y$ at the origin. Estimate the error in the approximation if $|x| \le 0.1$ and $|y| \le 0.1$.

19. Integrate $g(x,y,z) = x + y + z$ over the surface of the cube cut from the first octant by the planes $x = a, y = a, z = a$.

20. Find the surface area of a sphere of radius a.

SECTION - D

Answer any **2** questions among the questions 21 to 24. These questions carry **6** marks each. **(2×6=12)**

21. Find:

a) Unit tangent vector T ,

b) Unit normal vector N,

c) Curvature K,

d) Torsion τ and binomial vector B for the space curve $\vec{r}(t)=(3\sin t)\vec{i}+3(\cos t)\vec{j}+4t\vec{k}$.

22. Find the absolute maximum and minimum values of $f(x,y)=2+2x+2y-x^2-y^2$ on the triangular plate bounded by the lines $x=0, y=0, y=9-x$.

23. a) State both forms of Green's theorem.

b) Verify the circulation -curl form of Green's theorem for the field $\vec{F}(x,y)=(x-y)\vec{i}+x\vec{j}$ and the region R bounded by the unit circle .

$C:\vec{r}\ (t)=\cos t\vec{i}+\sin t\vec{j}, 0\leq t\leq 2\pi$

24. a) State Stoke's theorem.

b) Use Stoke's theorem to evaluate $\int_c \vec{F}.d\vec{r}$, if $F=xz\vec{i}+xy\vec{j}+3xz\vec{k}$ C is the boundary of the portion of the plane $2x+y+z=2$ in the first octant, traversed counter clock wise.

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K22U 2324

Reg. No.: ....

Name : ....

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2022
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
5B09MAT : Vector Calculus

Time : 3 Hours Max. Marks : 48

PART – A

(**Short Answer Questions**)

Answer **any four** questions from this Part. **Each** question carries **1** mark.

1. Find parametric equations for the line through (–2, 0, 4) parallel to $v = 2i + 4j - 2k$.

2. Find the distance from (1, 1, 3) to the plane $3x + 2y + 6z = 6$.

3. Find the gradient of the function $f(x, y) = x^2 + y^2$ at the point (1, –1).

4. Integrate $f(x, y, z) = x - 3y^2 + z$ over the line segment C joining the origin to the point (1, 1, 1).

5. One of the parametrization of the sphere $x^2 + y^2 + z^2 = 1$ is

PART – B

(**Short Essay Questions**)

Answer **any eight** questions. **Each** question carries **2** marks.

6. Find the curvature of the circle whose parametrization is given by $r(t) = (a \cos t)i + (a \sin t)j$.

7. Show that a moving particle will move in a straight line if the normal component of its acceleration is zero.

P.T.O.

8. A glider is soaring upward along the helix r(t) = (cos t)i + (sin t)j + tk. How long is the glider's path from t = 0 to t = 2 π ?

9. Find the directions in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ decreases most rapidly at (1, 1).

10. Suppose that a cylindrical can is designed to have a radius of 1 in. and a height of 5 in., but that the radius and height are off by the amounts dr = + 0.03 and dh = – 0.1. Estimate the resulting absolute change in the volume of the can.

11. Find the critical points of the function $f(x, y) = x^2 + y^2 - 4y + 9$.

12. Find the work done by the conservative field F = yzi + xzj + xyk, where f(x, y, z) = xyz, along any smooth curve C joining the point (–1, 3, 9) to (1, 6, –4).

13. Is the vector field $F = \frac{-y}{x^2 + y^2} i + \frac{x}{x^2 + y^2} j + 0k$ is conservative ? Justify your answer.

14. Find the divergence of the vector field $F = (y^2 - x^2)i + (x^2 + y^2)j$.

15. Integrate $G(x, y, z) = x^2$ over the cone $z = \sqrt{x^2 + y^2}, 0 \le z \le 1$.

16. Find the curl of F = xi + yj + zk.

PART – C

(**Essay Questions**)

Answer **any four** questions. **Each** question carries **4** marks.

17. Find the unit tangent vector of the curve $r(t) = (1 + 3 \cos t)i + (3 \sin t)j + t^2k$.

18. Find the angle between the planes 3x – 6y – 2z = 15 and 2x + y – 2z = 5.

19. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at the point (2, 0) in the direction of v = 3i – 4j.

20. Find $\frac{\partial w}{\partial x}$ if $w = x^2 + y^2 + z^2$ and $z^3 - xy + yz + y^3 = 1$.

21. Find the linearization $L(x, y, z)$ of $f(x, y, z) = x^2 - xy + 3 \sin z$ at the point $(2, 1, 0)$.

22. Verify Green's Theorem for the vector field $F(x, y) = (x - y)i + xj$ and the region R bounded by the unit circle $C : r(t) = (\cos t)i + (\sin t)j, 0 \leq t \leq 2\pi$.

23. Integrate $G(x, y, z) = xyz$ over the surface of the cube cut from the first octant by the planes $x = 1$, $y = 1$, and $z = 1$.

PART – D
(Long Essay Questions)

Answer **any two** questions. **Each** question carries **6** marks.

24. Find the curvature and torsion for the helix $r(t) = (a \cos t)i + (a \sin t)j + btk$, $a, b > 0, a^2 + b^2 \neq 0$.

25. Find the local extreme values of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$.

26. Show that $ydx + xdy + 4dz$ is exact and evaluate the integral $\int ydx + xdy + 4dz$ over any path from $(1, 1, 1)$ to $(2, 3, -1)$.

27. Find the surface area of the cone $z = \sqrt{x^2 + y^2}, 0 \leq z \leq 1$.

K21U 4554

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021
(2019 Admn. Only)
CORE COURSE IN MATHEMATICS
5B09MAT : Vector Calculus

Time : 3 Hours   Max. Marks : 48

PART – A

Short Answer

(Answer **any four**)   **(1×4=4)**

1. Find the parametric equations for the line through (3, 1, 2) and (2, 1, 6).

2. State and prove chain rule for vector functions.

3. Find the directional derivative of $F(x, y, z) = xy^2 - 4x^2y + z^2$ at (1, –1, 2) in the direction of 6i + 2j + 3k.

4. Evaluate $\oint_C x\,dx$ where C is the circle $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$.

5. If $F = (x^2y^3 - z^4)\, i + 4x^5y^2zj - y^4z^6k$ find curl F.

PART – B

Short Essay

(Answer **any eight**)   **(2×8=16)**

6. A helicopter is to fly directly from a helipad at the origin in the direction of the point (1,1,1) at a speed of 60 ft sec. What is the position of the helicopter after 10 sec ?

7. A projectile is launched from ground level with an initial speed $v_0 = 768$ ft/s. at an angle $\theta = 30°$. What is the range and maximum height attained by the projectile ?

8. Show that if $u = u_1 i + u_2 j + u_3 k$ is a unit vector, then the arc length parameter along the line $r(t) = (x_0 + tu_1)\, i + (y_0 + tu_2) j + (z_0 + tu_3)\, k$ from the point $P_0(x_0, y_0, z_0)$ where $t = 0$ is t itself.

9. Find the linearization of $f(x, y) = x^2 - xy + x^5y^2 + y^4$ at the point (2, 1).

10. Find the local extreme values of $f(x, y) = y^2 - x^2$.

11. Find the least squares line for the points (0, 1), (1, 3), (2, 2), (3, 4), (4, 5).

12. Prove the orthogonal gradient theorem.

13. A coil spring lies along the helix $r(t) = 2\cos t\; i + 2\sin t\; j + tk$, $0 \leq t \leq 2\pi$ with constant density δ. Find the spring's mass and center of mass, and its moment of inertia and radius of gyration about the z-axis.

14. Find work done by force $F = yz\; i + xz\; j + xy\; k$ acting along the curve given by $r(t) = t^3\; i + t^2 j + tk$ from t = 1 to t = 3.

15. Prove closed property of conservative fields.

16. Show that $ydx + xdy + 4dz$ is exact and evaluate the integral $\int ydx + xdy + 4dz$ over the line segment from (1, 1, 1) to (2, 3, – 1).

PART – C

Essay

(Answer **any four**) **(4×4=16)**

17. The position of a moving particle is given by $r(t) = 2\cos t\; i + 2\sin t\; j + 3tk$. Find the tangential, normal and binormal vectors. Also determine the curvature.

18. A delivery company accepts only rectangular boxes the sum of whose length and girth (perimeter of a cross-section) does not exceed 108 in. Find the dimensions of an acceptable box of largest volume.

19. Find the maximum and minimum values of the function $f(x, y) = 3x - y$ on the circle $x^2 + y^2 = 4$ using Lagrange multiplier's.

20. Given $F(x, y) = (y^2 - 6xy + 6)\ i + (2xy - 3x^2 - 2y)\ j$. Determine a potential function for F.

21. How are the constants a, b and c related if the following differential form is exact ? $(ay^2 + 2czx)\ dx + y(bx + cz)\ dy + (ay^2 + cz^2)\ dz$.

22. Calculate the outward flux of the field $F(x, y) = xi + 3y^2 j$ across the square bounded by the lines $x = \pm 2$ and $y = \pm 2$.

23. Find the surface area of sphere of radius a.

PART – D

Long Essay

(Answer **any two**) **(2×6=12)**

24. a) Show that the curvature of a circle with radius a is $^1/_a$.

    b) Find and graph the osculating circle of the parabola $y = x^2$ at the origin.

25. a) State Taylor's formula for f(x, y) at the origin and at point (a, b).

    b) Find a quadratic approximation to f(x, y) = sinx siny near the origin. How accurate is the approximation if $|x| \leq 0.2$ and $|y| \leq 0.4$?

26. a) Find the flux and circulation of the field $F(x, y) = (x - y)\ i + xj$ around the circle $x^2 + y^2 = 1$.

    b) Evaluate $\oint_C (x^2 - y^2)\ dx + (2y - x)\ dy$ where C consists of the boundary of the region in the first quadrant that is bounded by the graphs of $y = x^2$ and $y = x^3$.

27. a) Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2$, $z \geq 0$ by the cylinder $x^2 + y^2 = 1$.

    b) Evaluate $\oint_C zdx + xdy + ydz$ where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$. Orient C counter clockwise as viewed from above.

K21U 1535

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Sup./Imp.) Examination, November 2021
(2015 – '18 Admns)
CORE COURSE IN MATHEMATICS
5B08 MAT : Vector Calculus

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **four** questions are **compulsory**. **Each** carries **1** mark.

1. Find the gradient of the function $f(x, y) = e^{\frac{y}{x}}$ at (1, 1).

2. Find the divergence of $F(x, y) = 12x^2yi - 3(x^3 - y^2)\,j$ at (1, 1).

3. Define the line integral of a function f(x, y, z) over a smooth curve C.

4. Find a vector equation of the curve $y = 9 - x^2$, $y \geq 0$, in the plane. **(4×1=4)**

SECTION – B

Answer **any 8** questions from questions **5** to **14**. **Each** carries **2** marks.

5. Write the vector equation of a line in the plane passing through (1, 1) and making an angle $\frac{\pi}{6}$ with the positive X axis.

6. Find the distance travelled by a particle when it moves along the curve $r(t) = 3t^2i - \sqrt{2}t^2j + 5t^2k$, $0 \leq t \leq 3$.

7. If $\omega = x^2 + y^2 + z^2$ and $z^3 + xy + yz + y^3 = 0$, find $\frac{\partial\omega}{\partial x}$ at (x, y, z) = (2, 1, 0) treating x and y as independent variables.

8. Find the equation to the tangent plane to the surface $3xy^2 = 4yz + 2$ at (2, 1, 1).

9. If F(x, y) = C, show that $\frac{dy}{dx} = -\frac{\left(\frac{\partial F}{\partial x}\right)}{\left(\frac{\partial F}{\partial y}\right)}$.

P.T.O.

10. Find the linear approximation L(x, y) of the function $f(x, y) = (2x - y)^2$ at (1, 1).

11. Evaluate $\int_C (x + y)\, ds$ where C is given by $r(t) = ti + (1 - 3t)\, j$ from (0, 1) to (2, –5).

12. If the force $F = 3xi + 4yj + 3k$ is acting on a particle moving it along the curve C given by $r(t) = ti + (1 - t)j + tk$ from (0, 1, 0) to (1, 0, 1), find the work done by the force.

13. State the Gauss Divergence theorem.

14. Integrate $F(x, y, z) = x^2$ over the surface of the cone $z = \sqrt{x^2 + y^2}$, $0 \leq z \leq 1$. **(8×2=16)**

SECTION – C

Answer **any 4** questions from questions **15** to **20**. **Each** carries **4** marks.

15. Show that $\frac{d}{dt}(U \times V) = \frac{d}{dt}(U) \times V + U \times \frac{d}{dt}(V)$, where U, V are functions of t into $\mathbb{R}^2$.

16. Find the binormal vector to the curve $r(t) = \cos t\, i + \sin t\, j - k$ at $t = \frac{\pi}{4}$.

17. Suppose $D_U f(1, 2) = 10$ and $D_V f(1, 2) = 6$ where $U = \frac{3}{5}i - \frac{4}{5}j$ and $V = \frac{4}{5}i + \frac{3}{5}j$. Find $\frac{\partial f}{\partial x}(1, 2)$ and $\frac{\partial f}{\partial y}(1, 2)$.

18. Use the Taylor's formula for $f(x, y) = \cos(y + x)$ at the origin to find the quadratic approximation of f. Hence find approximate value of f(0.1, 0.2).

19. Find the scalar potential function of

$$F(x, y, z) = (y^2 \cos x + z^3)i + (2y \sin x - 4)\, j + 3xz^2 k.$$

20. Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane z = 4. **(4×4=16)**

SECTION – D

Answer **any 2** questions from questions **21** to **24**. **Each** question carries **6** marks.

21. If the equation of a curve is given by $r(t) = a \cos t\, i + a \sin t\, j + bt\, k$, find the unit tangent vector, unit normal vector and the curvature of the curve.

22. Using Lagrange's multiplier method, find the possible extreme points of $U(x, y) = x^2 + y^2$ subject to the constraint $x^2 + y^2 + 2x - 2y + 1 = 0$.

23. Verify Green's theorem for $\int_C (xy + y^2)\, dx + x^2 dy$ where C is the curve enclosing the region bounded by the parabola $y = x^2$ and the line $y = x$.

24. Verify Stoke's theorem for the vector field $F(x, y, z) = (2x - y)i - yz^2 j - y^2 z k$, taking the surface to be the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and the curve to be its boundary on the XY-plane. **(2×6=12)**

K20U 1535

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2020
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B08 MAT – Vector Calculus

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **four** questions are **compulsory**. **Each** question carries **1** mark :

1. Find the gradient of the function $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$ at (1, 1).
2. If $w = \sin(x + 2z)$ and $x^3 + z^3 = 3$, find $\frac{dw}{dx}$ using chain rule.
3. Evaluate $\int_C 9x^2y\,dx$ where C is given by $x = t^2$, $y = t^3$, $0 \le t \le 2$.
4. Write the formula for finding the surface area of a surface S given by $F(x, y, z) = C$, defined over the planar region R. **(4×1=4)**

SECTION – B

Answer **any 8** questions from questions **5** to **14**. **Each** question carries **2** marks :

5. Write the vector equation of a line in a plane passing through (1, 2) and making an angle $\frac{\pi}{3}$ with the positive X-axis.
6. Find the length of the curve $r(t) = 3\cos t\, i - 3\sin t\, j - 4t\, k$, $1 \le t \le 3$.
7. Find the directional derivative of $f(x, y) = e^{2xy}$ at (–2, 0) in the direction of $i + j$.

P.T.O.

8. If $w = x^2 + y^2 + z^2$ and $z^3 + xyz + yz^2 = 0$, find $\frac{\partial w}{\partial x}$ at $(x, y, z) = (1, 1, 0)$ treating x and y as independent variables.

9. Find the linear approximation $L(x, y)$ of the function $f(x, y) = e^{2y+x}$ at (2, 3).

10. If $\phi(x,y) = 3\sqrt{x^2+y^2}$, find div(grad($\phi$)).

11. Find the flux of $F = (x - y)i + xj$ across the circle $x^2 + y^2 = 1$ in the XY-plane.

12. If the force $F = 4xi + 4yj$ is acting on a particle moving it along the curve $r(t) = ti + (1 + 2t)j$ from (1, 3) to (3, 7), find the work done by the force.

13. Find a parametrization of the surface of the paraboloid $z = 16 - x^2 - y^2$, $z \geq 0$.

14. State the Gauss divergence theorem. **(8×2=16)**

SECTION – C

Answer **any 4** questions from questions **15** to **20**. **Each** question carries **4** marks :

15. Find the equation of the plane through the points A(1, 0, 2), B(1, 1, 1), C(1, 2, 3).

16. Show that $\frac{d}{dt}(U.V) = \frac{d}{dt}(U).V + U.\frac{d}{dt}(V)$, where U, V are functions of t into $\mathbb{R}^2$.

17. Use the Taylor's formula for $f(x, y) = e^x \cos y$ at the origin to find the quadratic approximation of f. Hence find approximate value of f(0.1, 0.2).

18. Find Curl $(F \times G)$ at (1, 2, 0) where $F(x, y, z) = 3x^2i + 2xyj + 2yzk$ and $G(x, y, z) = 4yzi + y^2j + xyz\,k$.

19. Using Green's theorem, find the area enclosed by the circle $x^2 + y^2 = 4$.

20. Evaluate the surface integral $\iint_\sigma y^2z^2\, dS$, where σ is the part of the cone $z = \sqrt{x^2+y^2}$ that lies between the planes $z = 1$ and $z = 2$. **(4×4=16)**

SECTION – D

Answer **any 2** questions from questions **21** to **24**. **Each** question carries **6** marks :

21. Find the unit tangent vector, unit normal vector and the binormal vector at $t = 0$ for the curve $r(t) = 2\cos t\, i + 2\sin t\, j + 4tk$.

22. Find the absolute maximum and minimum values of the function $f(x, y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices (0, 0), (3, 0) and (0, 5).

23. Check whether the vector field $F = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is conservative or not. If conservative, find its scalar potential function.

24. Verify Stoke's theorem for the vector field $F(x, y, z) = 2zi + 3xj + 5yk$, taking the surface to be the portion of the paraboloid $z = 4 - x^2 - y^2$ for $z \geq 0$, with upward orientation, and the curve to be the positively oriented circle of intersection of the paraboloid with the XY-plane. **(2×6=12)**

K19U 2258

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS- Sup./Imp.) Examination, November-2019

(2014-2016 Admissions)

Core Course in Mathematics

5B 09 MAT: GRAPH THEORY

Time : 3 Hours

Max. Marks : 48

SECTION - A

All the 4 questions are compulsory. They carry 1 mark each. **(4×1=4)**

1. Define self-complementary graphs.
2. State Cayley's formula.
3. What is meant by covering of a graph?
4. Define digraph.

SECTION - B

Answer any 8 questions among the questions 5 to 14. These questions carry 2 marks each. **(8×2=16)**

5. Prove that the sum of the degrees of the vertices of a graph is equal to twice the number of edges.
6. Let G be a simple graph. Prove that if G is disconnected then G^c is connected.
7. Let G be a connected graph and $e = xy$ be a cut edge of G. Prove that e does not belong to any cycle of G.
8. Determine the connectivity and edge-connectivity of the Petersen graph.
9. Prove that a tree with at least two vertices contains at least two pendant vertices.
10. Define branch, weight, centroid vertex of a vertex of a tree.
11. For any graph G with n vertices, prove $\alpha + \beta = n$.
12. Define Hamiltonian graphs and traceable graphs.
13. Define a symmetric digraph with an example.
14. Define in degree and out degree of a digraph with an example.

P.T.O.

SECTION - C

Answer any 4 questions among the questions 15 to 20. These questions carry 4 marks each. **(4×4=16)**

15. Explain with examples any four operations of graphs.

16. Prove that if G is a line graph, then $K_{1,3}$ is a forbidden subgraph of G.

17. Prove that a connected graph with at least two vertices contains at least two vertices that are not cut vertices.

18. Prove that every tree has a center consisting of either a single vertex or two adjacent vertices.

19. Determine the values of the parameters $\alpha, \beta, \alpha', \beta'$ for K_n.

20. Prove that a simple graph G with $n \geq 3$, if $d(u)+d(v) \geq n-1$ for every pair of non adjacent vertices u and v of G, then G is traceable.

SECTION - D

Answer any 2 questions among the questions 21 to 24. These questions carry 6 marks each. **(2×6=12)**

21. a) Define line graphs **(1)**

b) Let G_1 and G_2 be simple graphs. Prove that if G_1 and G_2 are isomorphic then $L(G_1)$ and $L(G_2)$ are isomorphic. **(3)**

c) Does the converse hold? Justify. **(2)**

22. a) Prove that for any loopless connected graph $k(G) \leq \lambda(G) \leq \delta(G)$. **(4)**

b) Determine $k(P)$ and $\lambda(P)$ for the Petersen graph P. **(2)**

23. For any non-trivial connected graph G, prove the following statements are equivalent. **(6)**

i) G is Eulerian.

ii) The degree of each vertex of G is an even positive integer.

iii) G is an edge-disjoint union of cycles.

24. a) Define tournaments and display all tournaments on three vertices. **(2)**

b) Prove that every tournament contains a directed Hamilton path. **(4)**

K21U 1536

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Sup.) Examination, November 2021
(2015 – 16 Admns)
Core Course in Mathematics
5B09MAT : GRAPH THEORY

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Define an edge cut of a graph G.

2. What is meant by branch of a vertex u of a tree T ?

3. What is clique ?

4. Define covering of a graph.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Prove that, if G is a simple graph with $\delta \geq \frac{n-1}{2}$, then G is connected.

6. Explain intersection and join of graphs with examples.

7. An edge e is a cut edge of a connected graph G, if and only if, there exists vertices u and v such that e belongs to every u – v path in G.

P.T.O.

8. Prove that if $m(G) = n(G)$ for a simple connected graph G, then G contains exactly one cycle.

9. If a graph G with at least three vertices is 2-connected, then show that any two vertices of G lie on a common cycle.

10. Prove that a tree with at least two vertices contains at least two pendant vertices.

11. Define matching. Distinguish between saturated and unsaturated set of vertices by a matching.

12. Show that a simple graph is a tree if and only if, any two vertices are connected by a unique path.

13. Explain the terms strict and symmetric digraphs with examples.

14. By sketching a wheel W_5, find a maximal covering and a minimal covering.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Show that a line graph of a simple graph G is a path if and only if, G is a path.

16. Prove that a connected graph G with at least two vertices contains at least two vertices that are not cut vertices.

17. Prove that if e is not a loop of G, then $\tau(G) = \tau(G - e) + \tau(G.e)$.

18. For any graph G, show that $\alpha + \beta = n$.

19. Write a short note on Tournament.

20. Prove that for a simple connected graph G, L(G) is isomorphic to G if and only if G is a cycle.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Prove that every tournament contains a directed Hamilton path.

22. Show that for a connected graph G with at least two vertices is a tree if and only if, its degree sequence $(d_1, d_2, ..., d_n)$ satisfies the condition : $\sum_{i=1}^{n} d_i = 2(n-1)$ with $d_i > 0$ for each i.

23. Prove that every vertex of a disconnected tournament with $n \geq 3$ is contained in a directed k-cycle, $3 \leq k \leq n$.

24. For a connected graph G, show that the following are equivalent.

   i) G is Eulerian.

   ii) The degree of each vertex of G is an even positive integer.

   iii) G is an edge disjoint union of cycles.

K20U 1536

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS-Sup.) Examination, November 2020
(2014 – 16 Admns.)
CORE COURSE IN MATHEMATICS
5B09MAT : Graph Theory

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Define a vertex cut of a graph G.
2. What is meant by weight of a vertex u of a tree T ?
3. When will you say that a graph is Eulerian ?
4. Define vertex independent of graphs. **(4×1=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Show that the sum of the degrees of the vertices of a graph is equal to twice the number of its vertices.
6. Distinguish between Cartesian product and normal product of graphs.
7. An edge e is a cut edge of a connected graph G, if and only if, e does not belong to any cycle of G. Prove it.
8. Show that a simple graph is a tree if and only if, any two vertices are connected by a unique path.
9. If $\delta(G) \geq 2$, for a graph G, then show that G contains a cycle.
10. If any two vertices of a graph G with at least three vertices, lie on a common cycle, then show that G is 2-connected.

P.T.O.

11. Prove that a subset S of the vertex set V of a graph is independent if V – S is a covering of G.
12. Distinguish between Eulerian and non Eulerian graphs with example.
13. Show that if a connected graph G is Eulerian, then the degree of each vertex is an even positive integer.
14. What is meant by orientation of a digraph ? Explain with an example. **(8×2=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Prove that the number of edges of a simple graph with ω components cannot exceed $\frac{(n-\omega)(n-\omega+1)}{2}$.
16. Show that the line graph of a simple graph G is a path if and if, G is a path.
17. Prove that a connected simple graph G is 3-edge connected if and only if, every edge of G is the intersection of the edge set of two cycles of G.
18. Show that every 2k-edge connected graph ($k \geq 1$) contains k pairwise edge disjoint spanning tree.
19. Discuss about vertex independence and edge independence with suitable examples.
20. Define degree of a vertex of a graph. Explain indegree and outdegree of a digraph with example. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Prove that if two simple graphs G_1 and G_2 are isomorphic, then $L(G_1)$ and $L(G_2)$ are isomorphic.
22. Prove that the number of edges of a connected graph with n vertices is n – 1 if and only if, it is a tree.
23. Show that for a graph G with $\delta > 0$, $\alpha' + \beta' = n$.
24. Prove that every tournament contains a directed Hamilton path. **(2×6=12)**

Reg. No. : ....

Name : ....

K19U 2259

V Semester B.Sc. Degree (CBCSS-Reg)
Examination, November - 2019
(2017 Admn. Only)
CORE COURSE IN MATHEMATICS
5B09 MAT- GRAPH THEORY

Time : 3 Hours Max. Marks : 48

SECTION - A

Answer **all** the questions. Each question carries **1** mark. **(4×1=4)**

1. What is the minimum number of edges of a simple connected graph on *n* vertices?
2. Define an Eulerian graph.
3. Draw $K_{2,3}$ and write the number of edges in that graph.
4. State Redei's theorem.

SECTION - B

Answer any **Eight** Questions. Each Question carries **2** marks **(8×2=16)**

5. Define Clique of a graph. Give one example.
6. Define a graphical sequence. Can (1, 1, 1, 2, 2) be a graphical sequence? Give reason.
7. Give example of a 3-regular graph. Is it possible to draw a 3-regular graph on 5 vertices ? Give reason.
8. Define Complement of a graph.
   Give example of a simple graph and its complement where both are connected.
9. If the edge $e = x - y$ of a connected graph G is a cut edge, then prove that e does not belong to any cycle of G.
10. In a tree, prove that any two distinct vertices are connected by a unique path.
11. Prove that a connected graph with n vertices and $n - 1$ edges is a tree.
12. Define a maximal independent set of vertices of a graph.

P.T.O.

13. If a graph is Eulerian, prove that the degree of each vertex of G is an even positive integer.
14. Give example of a graph that is Hamiltonian.

SECTION - C

Answer any **Four** Questions. Each Question carries **4** marks.**(4×4=16)**

15. Prove that in any group of n persons, $(n \geq 2)$ there are at least two with the same number of friends.
16. if a graph is bipartite, then prove that it contains no odd cycles.
17. Define the **union, intersection** and **join** of two graphs. Give one example for each.
18. A connected graph G with at least two vertices contains at least two vertices that are not cut vertices.
19. Prove that every connected graph contains a spanning tree.
20 In a graph G if each edge e belongs to an odd number of cycles of G, then prove that G is Eulerian.

SECTION - D

Answer any **Two** Questions. Each Question carries **6** marks. **(2×6=2)**

21. In a simple graph G with n vertices, if $\delta \geq \frac{n-1}{2}$, then prove that G is connected. What happens if the condition 'simple' is dropped ?
22. Prove that a vertex v of a connected graph G with at least three vertices is a cut vertex of G iff there exist vertices u and w of G distinct from v such that v is in every $u - w$ path in G.
23. If G is a connected labelled graph, what is the meaning of $\tau(G)$?
    If e is not a loop of a connected graph G, then prove that $\tau(G) = \tau(G-e) + \tau(Goe)$
24. For a non-trivial connected graph G, prove that the degree of each vertex of G is an even positive integer if and only if G is an edge-disjoint union of cycles.

K20U 1537

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2020
(2017 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B09 MAT : Graph Theory

Time : 3 Hours Total Marks : 48

PART – A

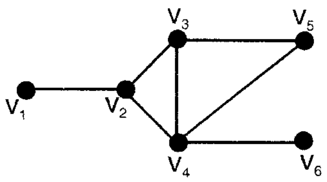
Answer **all 4** questions : **(4×1=4)**

1. Draw a graph on 4 vertices having a cut vertex. Mark the cut vertices.
2. Sketch 2 isomorphic trees on 4 vertices.
3. Plot a strict digraph on 4 vertices.
4. Sketch a symmetric digraph on 4 vertices.

PART – B

Answer **any 8** questions : **(8×2=16)**

5. Define a complete graph. Draw the graph K_5.
6. Picturise all non isomorphic graphs on 3 vertices.
7. If e = xy is a cut edge of a connected graph G, prove that there exist vertices u and v such that e belongs to every u-v path in G.
8. Find the cut edges and the cut vertices of the graph given below.



P.T.O.

9. Draw a 2 regular graph on 4 vertices and draw one spanning graph of the same.

10. For a connected graph G, define the terms diameter and eccentricity.

11. Find a covering and a minimal covering for the wheel graph W_5.

12. Give an example of an Eulerian graph. Explain why it is Eulerian.

13. Explain the terms Directed Walk and Directed Cycle.

14. Define the term tournament. Sketch a tournament on 3 vertices.

PART – C

Answer **any 4** questions : **(4×4=16)**

15. Plot all non isomorphic graphs on 4 vertices.

16. If a simple graph G is not connected, prove that G^c is connected.

17. Prove that a graph G with at least 3 vertices is 2-connected if and only if any two vertices of G lie on a common cycle.

18. Prove that a graph is a tree if and only if any two distinct vertices are connected by a unique path.

19. For a graph G on n vertices, define the terms independence number α and the covering number β of G. Further show that $\alpha + \beta = n$.

20. Describe Königsberg bridge problem. Represent the problem graphically. Does the problem has a solution ? Explain.

PART – D

Answer **any 2** questions : **(2×6=12)**

21. Show that a graph G is bipartite if and only if it contains no odd cycle.

22. Prove that a graph G with at least three vertices is 2-connected if and only if any two vertices of G are connected by at least 2 internally disjoint paths.

23. Establish the claim : A graph is Eulerian if and only if it has odd number of cycle decompositions.

24. Prove that every tournament contains a directed Hamiltonian path.
