K19U 2256

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS- Reg./Sup./Imp.) Examination, November-2019

(2014 Admn. Onwards)

Core Course in Mathematics

5B 07 MAT: Differential Equations, Laplace Transform and Fourier series

Time : 3 Hours Max. Marks : 48

SECTION - A

All the **4** questions are compulsory. They carry **1** mark each. **(4×1=4)**

1. Solve the differential equation $y'' = x^{-4}$.

2. Evaluate $(D-2)(D+1)\,e^{2x}$

3. Find the Laplace transform of $e^{t}\cosh 3t$

4. Show that if $f(x)$ and $g(x)$ have period p, then $h = af + bg$, where a and b are constants, has period p.

SECTION - B

Answer any **8** questions among the questions **5 to 14**. These questions carry **2** marks each. **(8×2=16)**

5. Show that $2xy\,dx + x^2dy = 0$ is exact and hence solve it.

6. Solve $y' - y = e^{2x}$.

P.T.O.

7. Solve the boundary value problem $y'' + y = 0, y(0) = 3, y(\pi) = -3$.

8. Define the Wronskian of two solutions y_1, y_2 of second order linear homogenous equation and find the Wronskian of e^x and xe^x.

9. Solve the non homogenous equation $y'' + 4y = 8x^2$.

10. Find a basis of solutions for $x^2y'' - xy + y = 0$, for positive x.

11. Define the unit step function and derive its Laplace transform.

12. State the convolution theorem and find the convolution of 1 and t.

13. Find the Fourier series of $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.

14. State the Fourier convergence theorem.

SECTION - C

Answer any **4** questions among the questions **15 to 20**. These questions carry **4** marks each. **(4×4=16)**

15. Give an example of an initial value problem, which has more than one solution.

16. State and prove the superposition principle for the homogenous linear system.

17. Solve $y'' + 10y' + 25y = e^{-5x}$.

18. Factor $p(D) = D^2 + D - 6$ and solve $p(D)[y] = 0$.

19. Find the inverse Laplace transform of $F(s) = \frac{2}{s^2} - \frac{2e^{-2s}}{s^2} - \frac{4e^{-2s}}{s} + \frac{se^{-\pi s}}{s^2+1}$.

20. Find the Fourier series of $f(x)=|x|, -2<x<2, f(x+4)=f(x)$.

SECTION - D

Answer any **2** questions among the questions **21 to 24**. These questions carry **6** marks each. **(2×6=12)**

21. Find the orthogonal trajectories of $y = cx^2$, where c is arbitrary.

22. Solve the differential equation $y'' + y = \sec x$.

23. Find the solution of $y''+2y'+2y=5u(t-2\pi)\sin t, y(0)=1, y'(0)=0$.

24. Find the Fourier series of $f(x)=\frac{x^2}{2}, -\pi<x<\pi, f(x+2\pi)=f(x)$. Hence show that $1-\frac{1}{4}+\frac{1}{9}-\frac{1}{16}+\ldots\ldots=\frac{\pi^2}{12}$ and $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\ldots\ldots=\frac{\pi^2}{6}$.

K21U 1534

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Sup./Imp.) Examination, November 2021
(2015-'18 Admns.)
CORE COURSE IN MATHEMATICS
5B07MAT : Differential Equations, Laplace Transform and Fourier Series

Time : 3 Hours Max. Marks : 48

PART – A

Answer **all 4** questions. **(1×4=4)**

1. Find an integrating factor of the differential equation xdy – ydx = 0.

2. Evaluate the Wronskian of y_1 = cost, y_2 = sint.

3. Write the Laplace transform of e^{at} cosbt.

4. Justify your answer the function f(x) = x^2 cosnx is even.

PART – B

Answer **any 8** questions. **(2×8=16)**

5. Solve the initial value problem y′ = –2xy, y(0) = 1.

6. Find the value of b for which the following equation is exact :

   $(xy^2 + bx^2y)dx + (x + y)\,x^2dy = 0.$

7. Obtain the differential equation associated with the primitive $y = Ax^2 + Bx + C$.

8. Find a particular solution of $y'' - 4y' - 4y = 2e^{2t}$.

9. Find the general solution of $(9D^2 - I)y = 0$, where D is the differential operator.

10. Write the Laplace transform of the function $f(t) = \begin{cases} e^t, & 0 < t < 1 \\ 0, & 1 < t < \infty \end{cases}$.

P.T.O.

11. Evaluate $L\left(\frac{1-e^t}{t}\right)$.

12. Find the inverse Laplace transform of the function $\frac{1}{s^2-4s+5}$.

13. Show that the product of two odd functions is even.

14. Sketch the graph of the function $f(x) = 1 - x^2$ if $-1 \le x \le 1$ and $f(x + 2) = f(x)$.

PART – C

Answer **any 4** questions. **(4×4=16)**

15. Solve the differential equation $y^2y' - y^3 \tan x = \sin x \cos^2 x$.

16. Given that Y_1 and Y_2 are solutions of the non-homogeneous equation $y'' + p(t)y' + q(t)y = g(t)$. Prove that $Y_1 - Y_2$ is a solution of the corresponding homogeneous equation $y'' + p(t)y' + q(t)y = 0$.

17. By method of variation of parameters, solve the differential equation, $y'' + y = \tan x$.

18. Assuming the required conditions, prove that $L[f'(t)] = sL[f(t)] - f(0)$.

19. Find the Fourier sine series expansion of $f(x) = 2 - x$ when $0 < x < 2$ with period 4.

20. Find the Fourier cosine integral representation of the function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$.

PART – D

Answer **any 2** questions. **(6×2=12)**

21. Find the orthogonal trajectories of the families of curves $\frac{1}{2}x^2 + y^2 = c$.

22. Using the method of indetermined coefficients, solve the differential equation $y'' - y = 2t^2$.

23. State and prove convolution theorem for Laplace transform.

24. Find the Fourier series of the function $f(x) = x^2$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.

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K22U 2323

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2022
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
5B08MAT : Differential Equations and Laplace Transforms

Time : 3 Hours Max. Marks : 48

PART – A
(**Short Answer**)

Answer **any four** questions from this Part. **Each** question carries **1** mark. **(4×1=4)**

1. Solve $dy + ydx = 0$.
2. State the order of the ODE $y'' + \pi y^3 = 0$.
3. Define Wronskian.
4. Write the characteristic equation of $\frac{d^3y}{dx^3} + y = \sin 4x$.
5. Define unit step function.

PART – B
(**Short Essay**)

Answer **any eight** questions from this Part. **Each** question carries **2** marks. **(8×2=16)**

6. Find the integrating factor of $ydx - xdy = 0$.
7. Find the order and degree of $\frac{d^3y}{dx^3} + 2\left(\frac{dy}{dx}\right)^{\frac{1}{2}} = 0$.
8. Show that a separable equation is also exact.
9. State the uniqueness theorem of first order differential equation.
10. Find the basis of the solution of the equation $\frac{d^2y}{dx^2} + y = 0$.

P.T.O.

11. Find the general solution of $\frac{d^2y}{dx^2} + 4y = 0$.

12. Write the standard form of Euler-Cauchy equation. Give one example of it.

13. Find the Wronskian of e^x and e^{-x}.

14. Find the convolution of t and e^{-t}.

15. Find the Laplace transform of $f(t) = t\cos 4t$.

16. Evaluate $L^{-1}\left[\frac{2}{(s+4)^3}\right]$.

PART – C
(**Essay**)

Answer **any four** questions from this Part. **Each** question carries **4** marks. **(4×4=16)**

17. Find the orthogonal trajectories of the family $y^2 = 2x^2 + c$.

18. Solve $(xy' + y = xy^{\frac{3}{2}}, y(1) = 4$.

19. Solve $\frac{d^2y}{dx^2} - 13\frac{dy}{dx} + 12y = e^{-2x}$.

20. Solve $\frac{d^2y}{dx^2} + 16y = -4\cos 4x$.

21. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^2$.

22. Find the Laplace transform of the function f(t) = t; if $t \geq 2$ and 0, if $t < 2$.

23. Solve $y'' + 3y' + 2y = r(t) = u(t-1) - u(t-2)$, $y(0) = 0$, $y'(0) = 0$.

PART – D
(**Long Essay**)

Answer **any two** questions from this Part. **Each** question carries **6** marks.

(2×6=12)

24. Solve $\left(\frac{3-y}{x^2}\right)dx + \left(\frac{y^2-2x}{xy^2}\right)dy = 0$, $y(-1) = 2$ by exactness.

25. Solve the initial value problem $\left(y + \sqrt{x^2 + y^2}\right)dx - xdy = 0,\ y(1) = 0$.

26. Solve $x^2y'' - 2xy + 2y = 0$, $y(1) = 1$, $y'(1) = 1$.

27. Using Laplace transform, solve $y'' + 4y' + 3y = e^{-t}$, $y(0) = y'(0) = 1$.

K22U 1963

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Supplementary) Examination, November 2022
(2016-18 Admissions)
CORE COURSE IN MATHEMATICS
5B07 MAT : Differential Equations, Laplace Transform and Fourier Series

Time : 3 Hours　　　　　　　　　　　　　　　　　　Max. Marks : 48

PART – A

Answer **all 4** questions. **(1×4=4)**

1. Verify whether the differential equation $(2x + 4y)dx + (2x - 2y)dy = 0$ is exact or not ?

2. Prove that the functions $e^{\pi t}$ and $\frac{1}{\pi}e^{\pi t}$ are linearly dependent.

3. State the second shifting theorem of Laplace of Laplace transform.

4. If f(x) is an odd function, the co-efficient of sines in the Fourier series expansion of f(x) is ___________

PART – B

Answer **any 8** questions. **(2×8=16)**

5. Solve the initial value problem $y' = (1-2x)y^2, y(0) = -\frac{1}{6}$.

6. State the existence and uniqueness theorem for first order initial value problems.

7. Given that Y_1 and Y_2 are solutions of the non-homogeneous equation $y'' + p(t)y' + q(t)y = g(t)$. Prove that $Y_1 - Y_2$ is a solution of the corresponding homogeneous equation $y'' + p(t)y' + q(t)y = 0$.

8. Find the general solution of $(D^2 + 2D + 5I)y = 0$, where D is the differential operator.

P.T.O.

9. Find a particular solution of $y'' - 2y' - 3y = 3e^{2t}$.

10. Find the Laplace transform of the function $f(t) = \begin{cases} \sin t, & 0 < t < \pi \\ 0, & \pi < t < \infty \end{cases}$.

11. Find $L(t^2e^{-3t})$.

12. Find the inverse Laplace transform of the function $\frac{4}{s^2 - 2s}$.

13. If f and g are periodic functions with same period T, show that any linear combinations of f and g is also T-periodic.

14. Sketch the graph of the function $f(x) = \begin{cases} 0, & -2 < x < 0 \\ 1, & 0 < x < 2 \end{cases}$ and $f(x + 4) = f(x)$.

PART – C

Answer **any 4** questions. **(4×4=16)**

15. Find the orthogonal trajectories of the families of curves $\frac{1}{2}x^2 + y^2 = c$.

16. Using the method of indetermined coefficients, solve the differential equation $y'' - y' - 2y = 6e^t$.

17. Find the general solution of $t^2y'' - 4ty' + 6y = 0$, $t > 0$.

18. Assuming the required conditions, prove that $L[f'(t)] = sL[f(t)] - f(0)$.

19. Show that the functions $\cos\left(\frac{\pi x}{L}\right)$ and $\sin\left(\frac{\pi x}{L}\right)$ are orthogonal.

20. Find the Fourier sine integral representation of the function $f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & x > 1 \end{cases}$.

PART – D

Answer **any 2** questions. **(6×2=12)**

21. Solve the differential equation $y^2y' - y^3 \tan x = \sin x \cos^2 x$.

22. By method of variation of parameters, solve the differential equation, $y'' + y = \sec x$.

23. Using Laplace transform, solve the initial value problem :

    $y'' - 3y' + 2y = 4e^{2t}$, given that $y(0) = -3$, $y'(0) = 5$.

24. Find the Fourier series of the function $f(x) = |x|$ if $-2 \le x \le 2$ and $f(x + 4) = f(x)$.

K20U 1534

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination, November 2020
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B07 MAT : Differential Equations, Laplace Transform and Fourier Series

Time : 3 Hours | Max. Marks : 48

PART – A

Answer **all 4** questions : **(4×1=4)**

1. Why the differential equation $y' + x^2y = \frac{1}{y}$ is linear ? Justify.
2. Find the Wronskian of $y_1 = e^{2t}$, $y_2 = e^{-3t}$.
3. Define Unit step function.
4. Show that the sum of two even functions is even.

PART – B

Answer **any 8** questions : **(8×2=16)**

5. Solve the differential equation $y' = (1 + x)(1 + y^2)$.
6. Check whether the equation $\cos(x + y)dx + (3y^2 + 2y + \cos(x + y))dy = 0$ is exact.
7. Solve the differential equation $y'' - 6y' + 9y = 0$.
8. Find a particular solution of $y'' - 2y' - 3y = 3e^{2t}$.
9. Find the general solution of $(D^2 + 3I)y = 0$, where D is the differential operator.

P.T.O.

10. Find the Laplace transform of the function $f(t)=\begin{cases}2, & 0<t<\pi \\ 0, & \pi<t<\infty\end{cases}$.

11. Find $L(te^{-t}\sin 3t)$.

12. Find the inverse Laplace transform of the function $\dfrac{1}{s(s^2+\omega^2)}$.

13. Sketch the graph of the function $f(x) = |x|$ if $-2 \leq x \leq 2$ and $f(x + 4) = f(x)$.

14. If f and g are periodic functions with same period T, show that any linear combinations of f and g is also T-periodic.

PART – C

Answer **any 4** questions : **(4×4=16)**

15. Solve the differential equation $xy' + y = xy^3$.

16. Given that Y_1 and Y_2 are solutions of the equation $y'' + p(t)y' + q(t)y = 0$. Prove that for any two constants c_1 and c_2, the linear combination $c_1Y_1 + c_2Y_2$ is also a solution for the differential equation.

17. Find the general solution of $t^2y'' - 4ty' + 6y = 0$, $t > 0$.

18. Assuming the required conditions, prove that $L[f'(t)] = sL[f(t)] - f(0)$.

19. Find the Fourier cosine series expansion of $f(x) = 2 - x$ when $0 \leq x \leq 2$ with period 4.

20. Find the Fourier integral representation of the function $f(x)=\begin{cases}1, & |x|<1 \\ 0, & |x|>1\end{cases}$.

PART – D

Answer **any 2** questions : **(2×6=12)**

21. Find the orthogonal trajectories of the families of curves $\frac{1}{2}x^2+y^2=c$.

22. By method of variation of parameters, solve the differential equation, $y'' - 5y' + 6y = 2e^t$.

23. State and prove convolution theorem for Laplace transform.

24. Find the Fourier series of the function $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.

K21U 4553

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021
(2019 Admn. Only)
CORE COURSE IN MATHEMATICS
5B08 MAT : Differential Equations and Laplace Transforms

Time : 3 Hours Max. Marks : 48

PART – A
(Short Answer)

Answer **any four** questions. **Each** question carries **1** mark.

1. Verify that $y = e^{2x^2}$ is a solution of the ODE $y' - 4xy = 0$.
2. Give an example of a first order nonlinear ODE.
3. Find a basis of solutions of the ODE $y'' - 4y = 0$.
4. What is Euler-Cauchy equations ?
5. State convolution theorem. **(4×1=4)**

PART – B
(Short Essay)

Answer **any eight** questions. **Each** question carries **2** marks.

6. Solve the initial value problem $y' = 6y$, $y(0) = 2$.
7. Does the initial value problem $xy' = y - 1$ has a unique solution ? Justify.
8. Solve the IVP $y' = -4x/y$, $y(2) = 3$.
9. Find the general solution of $y' + ky = e^{-kx}$.
10. Find the general solution of $4y'' - 25y = 0$.

P.T.O.

11. Factor $P(D) = D^2 - 3D - 40I$ and solve $P(D)y = 0$.

12. Find the general solution of $x^2y'' - 5xy' + 9y = 0$.

13. Find the Wronskian of cos 6x and sin 6x.

14. Find the inverse transform f(t) of $F(s) = \frac{e^{-s}}{s^2+4} + \frac{e^{-2s}}{s^2+1} + \frac{e^{-3s}}{(s+2)^2}$.

15. Find the Laplace transform of tsin2t.

16. Find the inverse transform of $\frac{1}{s(s^2+9)}$. **(8×2=16)**

PART – C

(Essay)

Answer **any four** questions. **Each** question carries **4** marks.

17. Solve the IVP $e^{2x}(2\cos y\, dx - \sin y\, dy) = 0$, $y(0) = 0$.

18. Find the general solution of $y' = 1/(6e^y - 2x)$.

19. Solve $y'' + y' = 0$ by reducing it to first order.

20. Solve the IVP $y'' + y' - 6y = 0$, $y(0) = 10$, $y'(0) = 0$.

21. Solve the nonhomogeneous ODE $y'' + y = \sec x$.

22. Find the Laplace transform of the function $f(t) = \begin{cases} 2, & \text{if } 0 < t < 1 \\ \frac{1}{2}t^2, & \text{if } 1 < t < \frac{1}{2}\pi \\ \cos t, & \text{if } t > \frac{1}{2}\pi. \end{cases}$

23. Solve the IVP $y'' + 3y' + 2y = \delta(t-1)$, $y(0) = 0$, $y'(0) = 0$ by Laplace transform. **(4×4=16)**

PART – D

(Long Essay)

Answer **any two** questions. **Each** question carries **6** marks.

24. Solve $2xyy' = y^2 - x^2$ by reducing it to variable separable form.

25. Solve the IVP $(e^{x+y} + ye^y)\, dx + (xe^y - 1)\, dy = 0$, $y(0) = -1$.

26. Solve the initial value problem $y'' - 6y' + 9y = e^{3x}$, $y(0) = 1$, $y'(0) = 1$.

27. Solve the integral equation $y(t) - \int_0^t (1+\tau)\, y(t\tau)\, d\tau = 1 - \sinh t$ by Laplace Transform.

(2×6=12)