**K19U 2255**

Reg. No. : ......................

Name : ..................................

**V Semester B.Sc. Degree (CBCSS- Reg./Sup./Imp.) Examination, November - 2019**

**(2014 Admn. Onwards)**

**Core Course in Mathematics**

**5B 06 MAT: ABSTRACT ALGEBRA**

Time : 3 Hours

Max. Marks : 48

## SECTION - A

Answer **All** Questions, Each question carries **One** Mark. **(4×1=4)**

1. Is the usual addition a binary operation on the set of all prime numbers? Justify your answer.
2. Define orbits of a permutation $\sigma$ of a set A.
3. Define normal subgroup of a group G. Given an example.
4. What is the characteristic of the ring of real numbers under usual addition and multiplication?

## SECTION - B

Answer any **Eight** Questions, Each question carries **Two** Marks. **(8×2=16)**

5. Prove that every cyclic group is abelian.
6. Show that a nonempty subset $H$ of group G is a subgroup of $G$ if and only if $ab^{-1} \in H$ for all $a, b \in H$.
7. Describe $S_n$, the symmetric group on $n$ letters.
8. Find the index of <3> in the group $Z_{24}$.

**P.T.O.**

9. Prove that the identity permutation in $S_n$ is an even permutation for $n \geq 2$.

10. Determine the number of group homomorphisms from $Z$ onto $Z$.

11. Find the characteristic of the ring $Z_6 \times Z_{21}$.

12. Prove that every field is an integral domain.

13. Define a ring and give an example of a finite ring which is not an integral domain.

14. Find the remainder of $8^{103}$ when divided by 13.

## SECTION - C

Answer any **Four** Questions, Each question carries **four** Marks.**(4×4=16)**

15. Show that every finite cyclic group of order $n$ is isomorphic to $< Z_n, +_n >$.

16. Show that the set of all permutations of any nonempty set A is group under permutation multiplication.

17. Prove that every group of prime order is cyclic.

18. Let $\varphi$ be a homomorphism of a group $G$ into $G'$. Then prove the following:

    a) If $a \in G$, prove that $\varphi(a^{-1}) = (\varphi(a))^{-1}$

    b) If $H$ is a subgroup of $G$, then $\phi[H]$ is a subgroup of $G'$

19. Prove that every finite integral domain is a field.

20. Prove that in the ring $Z_n$ the divisors of 0 are precisely those nonzero elements that are not relatively prime to $n$.

## SECTION - D

Answer any **Two** Questions, Each Question carries **Six** Marks.**(2×6=12)**

21. a) Define the greatest common divisor of two positive integers. Also find the quotient and remainder when 50 is divided by 8 according to division algorithm.

    b) Prove that subgroup of a cyclic group is cyclic.

22. a) State and prove Lagrange's Theorem.

b) Prove that the collection of all even permutations of $\{1,2,3,..........,n\}$ forms a subgroup of order $\frac{n!}{2}$ of the symmetric group $S_n; n \geq 2$.

23. a) State and prove the fundamental homomorphism theorem.

b) Show that a group homomorphism is one-one if and only if its kernel consists of only the identity element.

24. a) Show that the cancellation law holds in a ring iff it has no divisors of 0.

b) Show that the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is a divisor of 0 in $M_2(z)$.

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K20U 1533

Reg. No. : ...............................

Name : ...................................

# V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2020
# (2014 Admn. Onwards)
# CORE COURSE IN MATHEMATICS
# 5B06MAT : Abstract Algebra

Time : 3 Hours Max. Marks : 48

## SECTION – A

Answer **all** the questions **each** question carries **1** mark :

1. On $\mathbb{Z}^+$, define $*$ by letting $a * b = a^b$. Find $(2 * 2) * 3$.
2. What is the order of dihedral group $D_4$ ?
3. Let G be a group and let $\phi : G \to G$ by $\phi(g) = g^{-1}$. Is $\phi$ a homomorphism ?
4. Find the number of zero divisors of the ring $\mathbb{Z}_6$. **(4×1=4)**

## SECTION – B

Answer **any eight** questions **each** question carries **2** marks :

5. State and prove the left cancellation law of groups.
6. Find the remainder when – 61 is divided by 7.
7. Compute $\tau\sigma^2$, where $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$.
8. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$ into product cycles and transpositions.
9. Write all the left cosets of $4\mathbb{Z}$ of $\mathbb{Z}$.

P.T.O.

10. Prove that a group homomorphism $\phi : G \to G'$ is one-to-one map if and only if Ker ($\phi$) = {e}.

11. Find the order of $5 + \langle 4 \rangle$ in $\mathbb{Z}_{12}/\langle 4 \rangle$.

12. Let R be a ring with additive identity 0. Show that $(-a)(-b) = ab$, for any $a, b \in R$.

13. Define characteristic of a ring and give an example of a ring with characteristic 59.

14. Find all solutions of $2x \equiv 6 \pmod{4}$. **(8×2=16)**

SECTION – C

Answer **any four** questions **each** question carries **4** marks :

15. Prove that subgroup of a cyclic group is cyclic.

16. Show that every permutation on a finite set is a product of disjoint cycles.

17. State and prove Lagrange's theorem.

18. Show that a subgroup H of G is a normal subgroup if and only if $ghg^{-1} \in H$, for all $g \in G$ and $h \in H$.

19. Prove that every field is an integral domain.

20. If $a \in \mathbb{Z}$ and p is a prime not dividing a. Show that p divides $a^{p-1} - 1$. **(4×4=16)**

SECTION – D

Answer **any two** questions **each** question carries **6** marks :

21. Let G be a cyclic group with n elements and generated by a. Let $b \in G$ and let $b = a^s$. Prove that b generates a cyclic subgroup H of G containing n/d elements.

22. Prove that every group is isomorphic to a group of permutation.

23. State and prove the fundamental homomorphism theorem.

24. Let m be a positive integer and let $a, b \in \mathbb{Z}_m$. Let d be the gcd of a and b. Prove that the equation $ax = b$ has a solution in $\mathbb{Z}_m$ if and only if d divides b. **(2×6=12)**

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K22U 1962

Reg. No. : ...............................

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**V Semester B.Sc. Degree (CBCSS – Supplementary)**
**Examination, November 2022**
**(2016 – 18 Admissions)**
**CORE COURSE IN MATHEMATICS**
**5B06MAT – Abstract Algebra**

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions, **each** question carries **1** mark.

1. Define binary operation.
2. The order of the group $A_5$ is
3. Let $\phi : \mathbb{Z} \to \mathbb{R}$ under addition be given by $\phi(n) = n$. Find Ker ($\phi$).
4. A non-commutative division ring is called

SECTION – B

Answer **any eight** questions, **each** question carries **2** marks.

5. Let (G, $*$) be group. Show that $(a*b)' = b' * a'$, for all $a, b \in G$.
6. Write all subgroups of Klein-4 group.
7. Define orbits and find all the orbits of the permutation
$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix} \text{ in } S_8.$$
8. Define odd and even permutations and identify the permutation (1, 4, 5, 6) (2, 1, 5).
9. Does there exists a subgroup of order 20 of a group of order 30 ? Justify.
10. Let $\phi : G \to G'$ be a group homomorphism and let $a \in G$. Show that $\phi(a^{-1}) = \phi(a)^{-1}$.

P.T.O.

11. Write all the left cosets of $6\mathbb{Z}$ in $\mathbb{Z}$.
12. Let R be a ring with additive identity 0. Show that $a(-b) = (-a)\,b = -(ab)$, for any $a, b \in R$.
13. Find the solutions of the equation $x^2 - 5x + 6 = 0$ in $\mathbb{Z}_{12}$.
14. Compute the remainder of $8^{103}$ when divided by 13.

SECTION – C

Answer **any four** questions, **each** question carries **4** marks.

15. Find the cyclic subgroup of $\mathbb{Z}_{42}$ generated by 30.
16. In the permutation group $S_n$, show that the number of even and odd permutations are same.
17. Show that the order of an element of a finite group divides the order of the group.
18. Show that a subgroup H of G is a normal subgroup if and only if $gHg^{-1} = H$, for all $g \in G$.
19. In the ring $\mathbb{Z}_n$, show that the divisors of 0 are precisely those non zero elements that are not relatively prime to n.
20. Find all the solutions of the congruence $15x \equiv 27 \pmod{18}$.

SECTION – D

Answer **any two** questions, **each** question carries **6** marks.

21. Let G be a cyclic group with generator a. Prove that :
    a) If the order of G is infinite, then G is isomorphic to $(\mathbb{Z}, +)$
    b) If G has finite order n, then G is isomorphic to $(\mathbb{Z}_n, +_n)$.
22. State and prove the Cayley's theorem.
23. Let $\phi$ be a homomorphism of a group G into a group G′. Show that :
    a) If H is a subgroup of G, then $\phi[H]$ is a subgroup of G′.
    b) If K′ is a subgroup of G′, then $\phi^{-1}[K']$ is a subgroup of G.
24. Prove that the set $G_n$ of non zero elements of $\mathbb{Z}_n$ that are not 0 divisors forms a group under multiplication modulo n.

K22U 2322

Reg. No.: ..............................

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**V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2022**
**(2019 Admission Onwards)**
**CORE COURSE IN MATHEMATICS**
**5B07MAT : Abstract Algebra**

Time : 3 Hours | Max. Marks : 48

PART – A

Answer **any 4** questions. They carry **1** mark **each**.

1. Find the order of the cyclic subgroup of $\mathbb{Z}_4$ generated by 3.
2. What is the order of the cycle (1, 4, 5, 7) in $S_8$ ?
3. Let $\phi : G \to G'$ be a group homomorphism of G onto G′. If G is abelian, prove that G′ is abelian.
4. Let p be a prime. Show that $(a + b)^p = a^p + b^p$ for all a, $b \in \mathbb{Z}_p$.
5. Solve the equation 3x = 2 in the field $\mathbb{Z}_7$.

PART – B

Answer **any 8** questions from among the questions **6** to **16**. These questions carry **2** marks **each**.

6. Prove that in a group G, the identity element and inverse of each element are unique.
7. Let H and K be subgroups of a group G. Prove that $H \cap K$ is a subgroup of G.
8. State and prove division algorithm for $\mathbb{Z}$.
9. Let G be a group and suppose $a \in G$ generates a cyclic subgroup of order 2 and is the unique such element. Show that ax = xa for all $x \in G$.

P.T.O.

10. Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}$ and $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 1 & 3 & 6 & 5 \end{pmatrix}$ be permutations in $S_6$. Find $\tau\sigma$ and $|\langle\sigma\rangle|$.

11. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ in $S_8$ as a product of disjoint cycles and then as a product of transpositions.

12. Find all orbits of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 3 & 6 & 2 & 4 \end{pmatrix}$.

13. Find the index of $\langle 3 \rangle$ in the group of $\mathbb{Z}_{24}$.

14. Prove that every group of prime order is cyclic.

15. Prove that a group homomorphism $\phi : G \to G'$ is a one to one map if and only if $\ker(\phi) = \{e\}$.

16. Let R be a ring with additive identity 0. Then for any $a, b \in R$ prove that

    a) $a0 = 0a = 0$

    b) $a(-b) = (-a)b = -(ab)$.

## PART – C

Answer **any 4** questions from among the questions **17** to **23**. These questions carry **4** marks **each**.

17. Let G be a group and let g be one fixed element of G. Show that the map $I_g$, such that $i_g(x) = gxg'$ for $x \in G$ is an isomorphism of G with itself.

18. Draw subgroup diagram for Klein 4-group V.

19. Let G be a finite cyclic group of order n with generator a. Prove that G is isomorphic to $(\mathbb{Z}_n, +_n)$.

20. Let $n \geq 2$. Prove that the collection of all even permutations of $\{1, 2, 3, \ldots, n\}$ forms a subgroup of order $\frac{n!}{2}$ of the symmetric group $S_n$.

21. Let H be a subgroup of G such that $g^{-1}hg \in H$ for all $g \in G$ and all $h \in H$. Show that every left coset gH is the same as the right coset Hg.

22. Let H be a subgroup of G. Prove that left coset multiplication is well defined by the equation (aH) (bH) = (ab)H if and only if H is a normal subgroup of G.

23. Let $\phi : \mathbb{Z} \to S_8$ be homomorphism such that $\phi(1) = (1, 4, 2, 6)\ (2, 5, 7)$. Find $\ker(\phi)$ and $\phi(20)$.

PART – D

Answer **any 2** questions from among the questions **24** to **27**. These questions carry **6** marks **each**.

24. a) Let G be a cyclic group with n elements and generated by a. Let $b \in G$ and $b = a^s$. Prove that

    i) b generates a cyclic subgroup of H of G containing n/d elements, where d is the gcd of n and s.

    ii) $\langle a^s \rangle = \langle a^t \rangle$ if and only if gcd (s, n) = gcd (t, n).

    b) Let p and q be prime numbers. Find the number of generators of the cyclic group $\mathbb{Z}_{pq}$.

25. a) Prove that every coset (left or right) of a subgroup H of a group G has the same number of elements as H.

    b) State and prove Lagrange's theorem.

26. Let $\phi : G \to G'$ be a group homomorphism and let $H = \ker(\phi)$. Let $a \in G$. Prove that the set $\phi^{-1}[\{\phi(a)\}] = \{x \in G : \phi(x) = \phi(a)\}$ is the left coset aH of H and is also the right coset Ha of H.

27. a) Prove that every field F is an integral domain.

    b) Prove that every finite integral domain is a field.

    c) Give an example of an integral domain which is not a field.

K21U 1533

Reg. No. : ................................

Name : ...................................

# V Semester B.Sc. Degree (CBCSS – Sup./Imp.) Examination, November 2021 (2015 – '18 Admns) Core Course in Mathematics 5B06MAT : ABSTRACT ALGEBRA

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **1** mark.

1. True or false : A binary operation $*$ on a set S is commutative if there exists a, b $\in$ S such that $a * b = b * a$.
2. Cycle of length two is called ________________
3. Write a non-trivial improper normal subgroup of $S_3$.
4. Which element is the multiplicative inverse of 4 in the field $\mathbb{Z}_5$?

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

5. Let $*$ be a binary operation on $\mathbb{Q}^+$ defined by $a*b = \frac{ab}{2}$. Find the identity of $*$.
6. Let (G, $*$) be a group and if a, b $\in$ G. Show that the linear equation $a * x = b$ has a unique solution in G.
7. Define Octic group and write the elements.
8. Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 6 & 4 & 1 & 8 & 2 & 5 & 7 \end{pmatrix}$ into product of disjoint cycles.
9. Compute $(\langle 4 \rangle : \mathbb{Z}_{12})$.

P.T.O.

10. Let $\phi : G \to G'$ be a group homomorphism and e be the identity element in G. Show that $\phi(e)$ is the identity element of $G'$.

11. Define inner automorphism of a group.

12. Is $\mathbb{Z}$ a field ? Justify your claim.

13. Find all solutions of $x^2 + x = 2$ in the ring $\mathbb{Z}_4$.

14. Let R be a ring with unity and $n.1 \neq 0$ for all $n \in \mathbb{Z}^+$. Show that R has characteristic 0.

## SECTION – C

Answer **any four** questions. **Each** question carries **4** marks.

15. Prove that every cyclic group is abelian. What about the converse.

16. Write the composition table for the group $S_3$.

17. Prove that every group of prime order is cyclic.

18. Let $\phi : G \to G'$ be a group homomorphism and let $H = \mathrm{Ker}(\phi)$. Let $a \in G$. Show that the set $\phi^{-1}[\{\phi(a)\}] = \{x \in G \mid \phi(x) = \phi(a)\}$ is the left coset aH of H.

19. Show that the cancellation laws hold in a ring R if and only if R has no divisors of 0.

20. Show that for every integer n, the number $n^{33} - n$ is divisible by 15.

## SECTION – D

Answer **any two** questions. **Each** question carries **6** marks.

21. Find all subgroups of $\mathbb{Z}_{18}$ and draw the subgroup diagram.

22. Let G and $G'$ be groups and let $\phi : G \to G'$ be a one-to-one function such that $\phi(xy) = \phi(x)\phi(y)$ for all $x, y \in G$. Show that $\phi[G]$ is a subgroup of $G'$ and $\phi : G \to \phi[G]$ is an isomorphism.

23. Let H be a normal subgroup of a group G. Prove that the cosets of H form a group $G|H$ under the operation $(aH)(bH) = (ab)H$.

24. State and prove the Euler's theorem and find the remainder of $7^{1000}$ when divided by 24.

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K21U 4552

Reg. No. : ..................................

Name : ......................................

**V Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021**
**(2019 Admn. Only)**
**CORE COURSE IN MATHEMATICS**
**5B07 MAT : Abstract Algebra**

Time : 3 Hours Max. Marks : 48

## PART – A
**(Short Answer)**

Answer **any 4** questions. **Each** question carries **1** mark.

1. Define abelian group with an example.
2. Is $\mathbb{Z}^*$ under division a binary operation. Justify.
3. Every infinite order cyclic group is isomorphic to
4. What is the order of alternating group $A_n$ ?
5. State Lagrange's theorem. **(4×1=4)**

## PART – B
**(Short Essay)**

Answer **any eight** questions. **Each** question carries **2** marks.

6. In a group G with binary operation $*$, prove that there is only one element e in G such that $e * x = x * e = x, \forall x \in G$.
7. Prove that $(\mathbb{Q}^+, *)$, where $*$ is defined by $a * b = \frac{ab}{2}$; $a, b \in \mathbb{Q}^+$ is a group.
8. For sets H and K, Let $H \cap K = \{x/x \in H \text{ and } x \in K\}$, show that if H and K are subgroups of a group G, then $H \cap K$ is also a subgroup of G.

P.T.O.

9. Prove that the order of an element of a finite group divides the order of group.

10. Explain the elements of group $S_3$.

11. Find the order of (14)(3578) in $S_8$.

12. Prove that every permutation $\sigma$ of a finite set is a product of disjoint cycles.

13. Determine the permutation (18)(364)(57) in $S_8$ is odd or even.

14. State fundamental homomorphism theorem.

15. Find the order of $\mathbb{Z}_6/< 3 >$.

16. Let $\phi : G \to G'$ be a group homomorphism. Prove that Ker$\phi$ is a subgroup of G.

**(8×2=16)**

## PART – C

## (Essay)

Answer **any four** questions. **Each** question carries **4** marks.

17. Prove that subgroup of a cyclic group is cyclic.

18. Let G be a group and $a \in G$. Prove that $H = \{a^n / n \in \mathbb{Z}\}$ is the smallest subgroup of G that contains a.

19. Determine whether the set of all $n \times n$ matrices with determinant $-1$ is a subgroup of G.

20. Let A be a non-empty set. Prove that $S_A$, the collection of all permutations of A is group under permutation multiplication.

21. Define rings. Prove that $(\mathbb{Z}_n, +_n, \times_n)$ is a ring.

22. Prove that every group is isomorphic to a group of permutations.

23. Prove that $\gamma : \mathbb{Z} \to \mathbb{Z}_n$; where $\gamma(m) = r$ ; where r is the remainder when m is divided by n is a homomorphism.

**(4×4=16)**

## PART – D

## (Long Essay)

Answer **any two** questions. **Each** question carries **6** marks.

24. Prove that every integral domain is a field.

25. Let H be a subgroup of G, then prove that the left coset multiplication is well defined by the equation (aH)(bH) = (abH) if and only if H is a normal subgroup of G.

26. a) Find all cosets of the subgroup < 2 > of $\mathbb{Z}_{12}$.

    b) Prove that every group of prime order is cyclic.

27. Let G be a cyclic group with n elements generated by a. Let $b \in G$ and $b = a^s$, then prove that b generates a cyclic subgroup H of G containing $\frac{n}{d}$ elements, where d is the gcd of n and s. **(2×6=12)**

K22U 2320

Reg. No.: ..............................

Name : ..................................

**V Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, November 2022**
**(2019 Admission Onwards)**
**CORE COURSE IN MATHEMATICS**
**5B05MAT : Set Theory, Theory of Equations and Complex Numbers**

Time : 3 Hours Max. Marks : 48

PART– A

Answer **any four** questions from this Part. **Each** question carries **one** mark.

1. Give an example of a countable set.
2. Explain Descartes rule of signs.
3. If f(x) = 0 is an equation of odd degree, then it has at least one _________ root.
4. Say true or false. "Zero is a complex number".
5. Find the conjugate of 6 – 5i.

PART– B

Answer **any eight** questions from this Part. **Each** question carries **two** marks.

6. Define a denumerable set, give an example.
7. If $\alpha, \beta, \gamma$ are the roots of $2x^3 + x^2 - 2x - 1 = 0$, find
   i) $\alpha + \beta + \gamma$
   ii) $\alpha\beta\gamma$
   iii) $\alpha\beta + \beta\gamma + \alpha\gamma$.

P.T.O.

8. Search for rational roots of $f(x) = 2x^3 - 5x^2 + 5x - 3 = 0$.

9. Show that $x^5 - 2x^2 + 7 = 0$ has at least two imaginary roots.

10. Transform the equation $x^3 - 6x^2 + 5x + 12 = 0$, into an equation lacking second term.

11. Show that if $x = 1 + 2i$, then $x^2 - 2x + 5 = 0$.

12. Find the modulus and amplitude of $\sqrt{3} - i$.

13. Express $\frac{1+i}{2+3i}$ in the form of X + iY.

14. A) The solution of a reciprocal equation of first type depends on that of an reciprocal equation of first type and of __________ degree.

    B) The solution of a reciprocal equation of first type and of degree 2 m depends on that of an equation of degree ___________.

15. Find the roots of $2x^3 + 3x^2 - 1 = 0$.

16. A) Write the standard form of a cubic equation.

    B) What is reciprocal equation ?

## PART– C

Answer **any four** questions from this Part. **Each** question carries **four** marks.

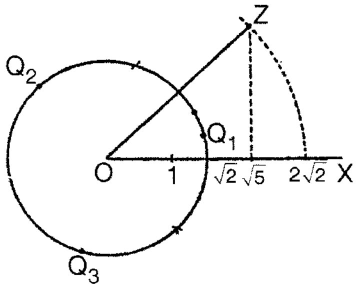
17. Show that the set $E_n = \{2n : n \in \mathbb{N}\}$ of even natural numbers is countably infinite.

18. If $\alpha, \beta, \gamma$ are the roots of $x^3 + P_1x^2 + P_2x + P_3 = 0$ then find the equation whose roots are $\alpha^3, \beta^3, \gamma^3$.

19. Find an upper bound and lower bound for the limit to the roots of $f(x) = 3x^4 - 61x^3 + 127x^2 + 220x - 520 = 0$.

20. Solve the reciprocal equation, $x^4 - 8x^3 + 17x^2 - 8x + 1 = 0$.

21. Find the points of $Q_1$, $Q_2$, $Q_3$ representing the values of $\sqrt[3]{z}$ where $z = \sqrt{5} + i\sqrt{3}$.




22. A) Define $n^{th}$ root of unity.

    B) Define Principal $n^{th}$ root of unity.

23. Explain the behaviour of roots of the equation $ax^3 + 3bx^2 + 3cx + d = 0$, with respect to discriminant.

PART– D

Answer **any two** questions from this Part. **Each** question carries **six** marks.

24. State and prove Cantor's theorem.

25. i) Find the condition that the sum of two roots of $\alpha$, $\beta$ of

    $x^4 + p_1x^3 + p_2x^2 + p_3x + P_4 = 0$, may be zero.

    ii) Use the result to find the roots of the equation, whose roots are the six values of $\frac{1}{2}(\alpha + \beta)$, where $\alpha$, $\beta$ are any roots of $ax^4 + 4bx^3 + 6cx^2 + 4dx + e = 0$.

26. If $\alpha$, $\beta$, $\gamma$ are the roots of $ax^3 + 3bx^2 + 3cx + d = 0$, then find the equation whose roots are squares of the difference of the roots.

27. Define multiplication and division of two complex numbers.

K21U 4550

Reg. No. : ...............................

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# V Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021
## (2019 Admn. Only)
## CORE COURSE IN MATHEMATICS
## 5B05 MAT : Set Theory, Theory of Equations and Complex Numbers

Time : 3 Hours Max. Marks : 48

### PART – A

Answer **any four** questions from this Part. **Each** question carries **1** mark.

1. State the Uniqueness theorem.
2. Sum of the roots of the equation $x^3 - x - 1 = 0$ is _______.
3. If $1 + i$ is a root of a quadratic equation, then the other root will be _______.
4. What is a reciprocal equation ?
5. If the discriminant $\Delta$ of a cubic equation is negative, then it has _______.

### PART – B

Answer **any eight** questions from this Part. **Each** question carries **2** marks.

6. If S is a finite set and $T \subseteq S$, then prove that T is finite.
7. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation which lacks the second term.
8. If $\alpha, \beta, \gamma$ are the roots of the equation $2x^3 + 3x^2 - x - 1 = 0$, then find the equation whose roots are $\alpha - 1, \beta - 1, \gamma - 1$.
9. State De Gua's rule.
10. Find an upper limit of the positive roots of the equation $x^3 - 10x^2 - 11x - 100 = 0$.
11. Write necessary and sufficient condition that the equation $ax^3 + 3bx^2 + 3cx + d = 0$ has two equal roots.
12. Discuss the character of the roots of the equation $x^3 + 29x - 97 = 0$ without finding them.
13. Explain the first and second kind reciprocal equations.
14. Express the complex number $2 + 2\sqrt{3}i$ in polar form.
15. Find $\text{Arg}(-1 - i)$.
16. State general form of De Movire's theorem.

P.T.O.

## PART – C

Answer **any four** questions from this Part. **Each** question carries **4** marks.

17. State and prove Cantor's theorem.

18. Use Descartes rule of signs to show that $x^7 - 3x^4 + 2x^3 - 1 = 0$ has at least four imaginary roots.

19. If $a + b + c = 0$, then show that $a^5 + b^5 + c^5 = 5abc\,(ab + bc + ca)$.

20. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

21. Solve $y^3 - 7y^2 + 36 = 0$, where the difference between two of the roots is five.

22. For any two complex numbers a and b, prove that
$$\left|a+\sqrt{a^2-b^2}\right|+\left|a-\sqrt{a^2-b^2}\right|=|a+b|+|a-b|\cdot$$

23. If $z = 1 + i$, then find $(1 + i)^{101}$.

## PART – D

Answer **any two** questions from this Part. **Each** question carries **6** marks.

24. Prove that the set of all rational numbers is denumerable.

25. Find the rational roots of the equation $x^3 - 5x^2 - 18x + 72 = 0$.

26. Explain the Cardan's solution for general cubic equation.

27. Find all the fourth roots of unity and locate them graphically.

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