K19U 3184

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CBCSS- Supplementary/Improvement) Examination, November-2019

(2017 -2018 Admissions)

CORE COURSE IN MATHEMATICS

1B01 MAT: DIFFERENTIAL CALCULUS

Time : 3 Hours   Max. Marks :48

SECTION-A

I. All the first **Four** questions are compulsory. They carry **1** mark each. **(4×1=4)**

1. Find $\lim_{y\to 2}\frac{y+2}{y^2+5y+6}$.

2. Find the value of $\cosh x$ if $\sinh x=\frac{4}{3}$

3. Find the Cartesian coordinate corresponding to $(-3,\pi)$.

4. Find an equation for the circular cylinder $4x^2+4y^2=9$ in cylindrical coordinates.

SECTION - B

II. Answer any **Eight** questions from among the questions **5** to **14.** These questions carry 2 marks each. **(8×2=16)**

5. Evaluate $\lim_{u\to 1}\frac{u^4-1}{u^3-1}$.

6. Show that $f(x)=\frac{x^2+x-6}{x^2-4}$ has a continuous extension to $x=2$, and

find that extension.

7. Find the derivative of $y = \sinh^{-1}(\tan x)$ with respect to x.

8. Find the spherical coordinate equation for $x^2 + y^2 + \left(z - \frac{1}{2}\right)^2 = \frac{1}{4}$.

9. Calculate $\frac{dS}{d\theta}$ for $r = a(1 - \cos\theta)$.

10. Find the radius of curvature of the parabola $y^2 = 4ax$ at $(at^2, 2at)$

11. Evaluate $\lim\limits_{x \to \frac{\pi}{2}^-} \frac{\sec x}{1 + \tan x}$

12. Find the maximum and minimum values of $3x^4 - 2x^3 - 6x^2 + 6x + 1$ in the interval $(0,2)$.

13. Find an equation for the level surface of the function $f(x,y,z) = \ln(x^2 + y + z^2)$ through (-1,2,1).

14. Show that the function $f(x,y) = \frac{2x^2y}{x^4 + y^2}$ has no limit as (x,y) approaches (0,0).

SECTION - C

III. Answer any **Four** questions from among the questions **15 to 20.** These questions carry **4** marks each. **(4×4=16)**

15. If $x = a(\cos t + t\sin t), y = a(\sin t - t\cos t)$. find $\frac{d^2y}{dx^2}$.

16. For the cardiod $r = a(1 + \cos\theta)$ show that $\frac{\rho^2}{r}$ is constant.

17. Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e 1 . 1$ correct

to 4 decimal places.

18. Verify Langrange's mean value theorem for
$f(x) = (x-1)(x-2)(x-3)$ in $(0,4)$ and find appropriate value for c.

19. Express $\frac{\partial \omega}{\partial r}$ and $\frac{\partial \omega}{\partial s}$ in terms of r and s,

if $\omega = x + 2y + z^2, x = \frac{r}{s}, y = r^2 + \ln s, z = 2r$.

20. Verify Euler's theorem for $z = (x^2 + xy + y^2)^{-1}$.

SECTION - D

IV. Answer any **Two** questions from among the questions **21** to **24**. These questions carry **6** marks each. **(2×6=12)**

21. If $y = e^{a \sin^{-1} x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$ .Hence find the value of y_n when $x=0$.

22. Find the coordinates of the center of curvature at the point $x=at^2$, $y=2at$. on the parabola $y^2=4ax$ and hence find its evolute.

23. Find the volume of the largest possible right circular cylinder that can be inscribed in a sphere of radius a.

24. If $u = \tan^{-1} x\left(\frac{x^3+y^3}{x-y}\right), x \neq y$ show that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1-4\sin^2 u)\sin 2u$.

K19U 3320

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree CBCSS(OBE) - Regular Examination, November - 2019
(2019 Admissions)
Core Course in Mathematics
1B01MAT : SET THEORY, DIFFERENTIAL CALCULUS AND NUMERICAL METHODS

Time : 3 Hours   Max. Marks : 48

Part - A

Answer any 4 questions. Each Question carries 1 mark.

1. Find $gof(2)$ if $f: R \to R$ and $g: R \to R$ are given by $f(x)=3x-1$; $g(x)=x^2-2$.

2. Find the limit $\lim_{x\to 0}\frac{\tan x}{x}$.

3. Find $\frac{\partial^2 z}{\partial x \partial y}$, if $z=\sin(2x-3y)$.

4. Find the degree of the homogenous equation $f(x,y)=\frac{\sqrt{x}}{x^2+y^2}$.

5. Is the relation R ={(1,1), (1,2), (2,2), (1,3), (2,3), (3,3), (2,1)} a partial order on {1,2,3}? Justify.

Part - B

Answer any 8 questions. Each question carries 2 marks.

6. Check if the function $f: R \to R$ given by $f(x)=\frac{3x+1}{2}$ is one-to-one and onto.
7. Define equivalence relation and check if R={(1,1), (1,2), (2,2), (3,3), (1,3)} on the set A = {1,2,3} is an equivalence relation or not.
8. Give an example of a function $f: R \to R$ which is one-to-one but not onto.
9. Write the inverse relation R^{-1}, if the relation R on the set A = {1,2,3,4,5} is given by R={(m,n):m'divide'n}
10. Find the symmetric closure and reflexive closure of the relation on the set A = (1,2,3,4) given by R={(1,1), (1,2), (1,3), (2,3)}.

P.T.O.

11. Find the limit $\lim\limits_{x\to 0}\frac{x}{|x|}$, if exists. Justify your answer.

12. Find the points of discontinuity of the function $f(x)=\frac{x+2x^2}{x^2-4x+3}$, if any.

13. If a>0, $a\le f(x)\le a+x$, $a-x\le g(x)\le a$ and both the limits $\lim\limits_{x\to 0} f(x), \lim\limits_{x\to 0} g(x)$ exist, find $\lim\limits_{x\to 0}\frac{f(x)}{g(x)}$.

14. If $z=u^2+v^2$ and $u=at^2$ $v=2at$, find $\frac{dz}{dt}$ using chain rule.

15. If $x^3+3x^2y+6xy^2+y^3=1$, find $\frac{dy}{dx}$.

16. Find a root of $xe^x-2=0$ using bisection method.

Part - C

Answer any 4 questions. Each question carries 4 marks.

17. Show that $f: R\to R$ defined by $f(x)=\frac{ax+b}{c}, a\neq 0, c\neq 0$ is one-to-one and onto. Find formula for f^{-1}.

18. Give an example of a function $f: R\to R$ whose limit does not exists at any point of R (with justifications)

19. Evaluate the following limits:

i) $\lim\limits_{x\to 3}\frac{x^3-27}{x^4-81}$

ii) $\lim\limits_{h\to 0}\frac{\sqrt{5h+4}-2}{h}$

20. Examine the continuity of the function $f(x,y)=\begin{cases}\frac{3x-y}{2x+y} & if (x,y)\neq(0,0)\\ 0, & if (x,y)=(0,0)\end{cases}$ at the points (0,0) and (1,0).

21. If $u = e^x \cos(y)$, $v = e^x \sin(y)$ and $f(x, y)$ is any function of x and y, then show that

i) $\dfrac{\partial f}{\partial x} = u\dfrac{\partial f}{\partial u} + v\dfrac{\partial f}{\partial v}$

ii) $\dfrac{\partial f}{\partial y} = -v\dfrac{\partial f}{\partial u} + u\dfrac{\partial f}{\partial v}$

22. Show that if $y = f(x+at) + g(x-at)$ with f and g twice differentiable, then $\dfrac{\partial^2 y}{\partial t^2} = a^2 \dfrac{\partial^2 y}{\partial x^2}$.

23. Derive the Newton-Raphson formula for finding the root of an equation.

Part - D

Answer any 2 questions. Each question carries 6 marks.

24. Let A = {1, 2, 3, ..., 9, 10}. The relation '~' on A x A is defined by $(a, b) \sim (c, d)$ if $ad = bc$. Check whether this is an equivalence relation. If so, find the equivalence classes [(1,1)], [(1,3)] and [(3,1)].

25. If $y = e^{a \sin^{-1} x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. Hence find the value of y_n when $x=0$.

26. If $u = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = m(m+1)r^{m-2}$.

27. Find the point of intersection of the curve $y=x^3$ and the line $y=3x-1$ using regula-falsi method, starting with suitable initial approximations (correct to two decimal places).

K20U 3320

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree CBCSS (OBE) – Reg./Supple./Improve. Examination, November 2020
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
1B01 MAT : Set Theory, Differential Calculus and Numerical Methods

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question is of **1** mark.

1. Find gof(2) if $f : R \to R$ and $g : R \to R$ are given by $f(x) = 3x - 1$; $g(x) = x^2 - 2$.
2. Find the limit $\lim\limits_{x \to 0} \frac{\tan x}{x}$.
3. If $z = xyf(x/y)$, find the value of 'n' such that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = (n-1)z$.
4. Find the first order partial derivatives of $z = e^{-x+y}$.
5. Is the relation $R = \{(1, 1), (1, 2), (2, 2)\}$ a partial order on $\{1, 2\}$? Justify your answer.

PART – B

Answer **any 8** questions. **Each** question is of **2** marks.

6. Check if the function $f : R \to R$ given by $f(x) = \frac{3x+1}{2}$ is one-to-one and onto.
7. Check if the relation $f = \{(2, 3), (1, 4), (2, 1), (3, 2), (4, 3)\}$ from $A = \{1, 2, 3, 4\}$ to itself is a function or not.
8. On the set $A = \{1, 2, 3, 4, 5, 6\}$, consider the relation

   $R = \{(1, 1), (1, 5), (2, 2), (2, 3), (2, 6), (3, 2), (3, 3), (3, 6), (4, 4), (5, 1), (5, 5), (6, 2), (6, 3), (6, 6)\}$. Find the partitions of A induced by R.

P.T.O.

9. Define equivalence relation and check if R = {(1, 1), (1, 2), (2, 2), (3, 3), (1, 3)} on the set A = {1, 2, 3} is an equivalence relation or not.

10. Give an example of a relation on A = {1, 2, 3, 4, 5} which is both an equivalence relation and a partial order on it. Justify.

11. Is the function $f(x,y) = \begin{cases} \frac{x^2-1}{x-1}, \text{ if } x \neq 1 \\ 2, \text{ if } x = 1 \end{cases}$ continuous at x = 1 ? Justify your answer.

12. Find the limit $\lim\limits_{x \to 0} \frac{x}{|x|}$, if exists. Justify your answer.

13. If $a > 0$, $a \leq f(x) \leq a + x$, $a - x \leq g(x) \leq a$ and both the limits $\lim\limits_{x \to 0} f(x)$, $\lim\limits_{x \to 0} g(x)$ exist, find $\lim\limits_{x \to 0} \frac{f(x)}{g(x)}$.

14. If $z = u^2 + v^2$ and $u = at^2$, $v = 2at$, find $\frac{dz}{dt}$ using chain rule.

15. Find $\frac{dy}{dx}$ if $x^y = y^x$.

16. Find a root of $xe^x - 2 = 0$ using bisection method.

PART – C

Answer **any 4** questions. **Each** question is of **4** marks.

17. On the set of all natural numbers N, define a relation R by (a, b) ∈ R if '6 divides a – b'. Show that the relation is an equivalence relation and write down the collection of all equivalence classes.

18. Show that for a function f, $\lim\limits_{x \to c} |f(x)| = 0$ implies $\lim\limits_{x \to c} f(x) = 0$. Is it true that $\lim\limits_{x \to c} |f(x)| = 1$ implies $\lim\limits_{x \to c} f(x) = 1$? Justify.

19. Evaluate the following limits :

i) $\lim\limits_{x \to 3} \frac{x^3 - 27}{x^4 - 81}$

ii) $\lim\limits_{h \to 0} \frac{\sqrt{5h+4} - 2}{h}$.

20. For $f(x,y) = \frac{3x-y}{2x+y}$, find the limits $\lim\limits_{x\to 0}(\lim\limits_{y\to 0} f(x,y))$ and $\lim\limits_{y\to 0}(\lim\limits_{x\to 0} f(x,y))$. Is this function continuous at (0, 0) ? Justify your claim.

21. If $u = e^x \cos(y)$, $v = e^x \sin(y)$ and f(x, y) is any function of x and y, then show that

i) $\frac{\partial f}{\partial x} = u\frac{\partial f}{\partial u} + v\frac{\partial f}{\partial v}$

ii) $\frac{\partial f}{\partial y} = -v\frac{\partial f}{\partial u} + u\frac{\partial f}{\partial v}$.

22. Show that if $y = f(x + at) + g(x - at)$ with f and g twice differentiable, then $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$.

23. Determine the root of $x^4 + x^3 - 7x^2 - x + 5 = 0$, which lies in between 2 and 3 using Regula-falsi method, correct to three decimal places.

PART – D

Answer **any 2** questions. **Each** question is of **6** marks.

24. Let $f : A \to B$, $g : B \to C$ be two functions. Prove that

i) If both f and g are one-to-one, then gof is also one-to-one.

ii) If both f and g are onto, then gof is also onto.

iii) If gof is one-to-one, then f must be one-to-one.

25. If $y = e^{a\sin^{-1}x}$, prove that $(1 - x^2)\, y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$. Hence find the value of y_n when x = 0.

26. If $u = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = m(m+1)r^{m-2}$.

27. Use Newton-Raphson method to find the root of $x^4 - x - 10 = 0$ which is near to 2, correct to three decimal places.

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K19U 3183

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CBCSS-Supplementary)
Examination, November - 2019
(2014-2016 Admissions)
CORE COURSE IN MATHEMATICS
1B01 MAT : DIFFERENTIAL CALCULUS

Time : 3 Hours Max. Marks : 48

SECTION - A

All the first **Four** questions are compulsory. They carry **1** mark each.

1. Evaluate $\lim\limits_{x \to -1} \frac{x^3 + 4x^2 - 3}{x^2 + 5}$.

2. Find the value of $\cosh\frac{x}{3}$ if $\sinh\frac{x}{3} = \frac{4}{3}$.

3. Find the Cartesian coordinates corresponding to $(2, \frac{\pi}{2})$.

4. Transform the equation $x^2 + y^2 - 3z^2 = 0$ to spherical coordinates.

SECTION - B

Answer any **8** questions from among the questions **5 to 14**. These questions carry **2** marks each.

5. Evaluate $\lim\limits_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$.

P.T.O.

6. Suppose $\lim_{x\to c} f(x) = 5$ and $\lim_{x\to c} g(x) = -2$. Find $\lim_{x\to c} f(x)g(x)$ and $\lim_{x\to c} f(x) + 3g(x)$.

7. Prove that $\frac{d}{dx}\cosh^{-1}x = \frac{1}{\sqrt{x^2-1}}$.

8. Find the center and radius of the sphere whose equation is $x^2 + y^2 + z^2 - \frac{2}{3}x - y - \frac{4}{3}z - \frac{22}{3} = 0$.

9. The generators of a cylinder are parallel to the line $6x = -3y = 2z$ and the guiding curve is given by $x^2 + y^2 = 1, z = 0$. Find its equation.

10. Verify Rolle's theorem for $f(x) = (x+2)^3(x-3)^4$ in(-2,3).

11. Using Maclaurin's series, expand log $(1+x)$.

12. Evaluate $\lim_{x\to 0^+} x\cot x$.

13. Find domain and the range of $w = \frac{1}{x^2 + y^2 + z^2}$.

14. Show that $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ is continuous at every point except at the origin.

SECTION - C

Answer any **Four** questions from among the questions **15 to 20**. These questions carry **4** marks each.

15. Show that $\lim_{x\to 1}(5x-3)=2$.

16. Verify Langrange's mean value theorem for $f(x)=(x-1)(x-2)(x-3)$ in (0.4) and find appropriate value for c.

17. Show that the radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ is $4a\cos\frac{\theta}{2}$.

18. Find the asymptotes of the curve of $x^3+3x^2y-4y^3-x+y+3=0$.

19. Find the linearization $L(x,y,z)$ of $f(x,y,z)=x^2-xy+3\sin z$ at the point (2,1,0).

20. Verify Euler's theorem for $z=(x^2+xy+y^2)^{-1}$.

SECTION - D

Answer any **2** questions from among the questions **21 to 24**. These questions carry **6** marks each.

21. If $y=\sin(m\sin^{-1}x)$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(m^2-n^2)y_n=0$ also find $(y_n)_o$.

22. Show that the points (-4,3,6),(-5,2,2),(-7,6,6) and (-8,5,2) are concyclic.

23. Show that the evolute of the cycloid $x=a(t+\sin t), y=a(1-\cos t)$ is the curve $x=a(t-\sin t), y-2a=a(1+\cos t)$.

24. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, $x\neq y$ show that.

i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(1-4\sin^2 u)\sin 2u.$

K20U 3184

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CBCSS – Supplementary) Examination, November 2020 (2017-2018 Admissions) CORE COURSE IN MATHEMATICS 1B01 MAT : Differential Calculus

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each** :

1. Find $\lim_{t \to 6} 8(t-5)(t-7)$.

2. Find $\frac{d}{dx} \sinh \frac{x}{3}$.

3. Find the Cartesian equation for $r^2 = 4r \cos\theta$.

4. Find an equation for the cylinder $x^2 + (y-3)^2 = 9$ in cylindrical coordinates.

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each** :

5. Evaluate $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$.

6. For what value of a is $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$ continuous at every x = 2.

7. Express $\sinh^{-1}\left(\frac{-5}{12}\right)$ in terms of natural logarithms.

P.T.O.

8. Find the Cartesian equation for $\rho = 5 \cos\phi$.

9. Find $\frac{dS}{d\theta}$ for the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

10. Find the radius of curvature of the catenary $y = c \cosh\left(\frac{x}{c}\right)$ at (0, c).

11. Verify Lagrange's mean value theorem for $f(x) = \log_e x$ in [1, e] and find appropriate value for c.

12. Find $\lim\limits_{x \to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$.

13. Find $\lim\limits_{(x, y) \to (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

14. Find the value of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point (4, –5) if $f(x, y) = x^2 + 3xy + y - 1$.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each** :

15. If $ax^2 + 2hxy + by^2 = 1$, prove that $\frac{d^2y}{dx^2} = \frac{h^2 - ab}{(hx + by)^3}$.

16. Find the radius of curvature at the point (r, θ) on the curve $r^n = a^n \cos n\theta$.

17. Using Maclaurin's series, expand sinx.

18. Verify Rolle's theorem for $f(x) = (x + 2)^3(x - 3)^4$ in (–2, 3).

19. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s, if $w = x^2 + y^2$, $x = r - s$, $y = r + s$.

20. Verify Euler's theorem for $z = ax^2 + 2hxy + by^2$.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each** :

21. If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$ prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

22. Find the evolute of the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$.

23. Find the volume of the largest possible right circular cylinder that can be inscribed in a sphere of radius a.

24. If $u = \dfrac{x^2y^2}{x^2 + y^2}$, show that

    i) $x\dfrac{\partial^2 u}{\partial x^2} + y\dfrac{\partial^2 u}{\partial y \partial x} = \dfrac{\partial u}{\partial x}$

    ii) $x\dfrac{\partial^2 u}{\partial x \partial y} + y\dfrac{\partial^2 u}{\partial y^2} = \dfrac{\partial u}{\partial y}$

    iii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = 2u$.

K21U 6798

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CBCSS – O.B.E. – Regular/Supplementary/ Improvement) Examination, November 2021
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
1B01MAT : Set Theory, Differential Calculus and Numerical Methods

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions from this Part. **Each** question carries **1** mark.

1. Give an example of an antisymmetric relationship.
2. Consider $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 + 1$. Find f ([–1, 1]).
3. State the intermediate value theorem for continuous functions.
4. Find the domain of the real valued function $f(x, y) = \sqrt{y - x - 2}$
5. For $z = x^2y - y\cos x$, find $\frac{\partial z}{\partial x}$.

PART – B

Answer **any 8** questions from this Part. **Each** question carries **2** marks.

6. Find the domain of the real valued function $f(x) = \sqrt{x^2 - 5x + 6}$.
7. Using arithmetic modulo M = 11, evaluate 2 – 5.
8. Give an example of a function which is not one-to-one.
9. If $\sqrt{9 - 2x} \le f(x) \le \sqrt{9 - x^2}$ for $-1 \le x \le 1$, then find $\lim_{x \to 0} f(x)$.
10. If $\lim_{x \to 2} \frac{f(x)}{x^2} = 1$, find $\lim_{x \to 2} \frac{f(x)}{x}$.
11. If y = sin(sin x), prove that $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$.
12. Show that the function w = sin (x + ct) is a solution of the wave equation $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$.
13. Find $\frac{\partial z}{\partial x}$ where $yz - \ln z = x + y$ defines z as a function of x and y.

P.T.O.

14. Find all second order partial derivatives of the function $z = \frac{x}{x-y}$.

15. State Euler's theorem on homogeneous functions.

16. Determine the maximum number of positive and negative roots of the equation $3x^3 - x^2 - 10x + 1 = 0$.

PART – C

Answer **any 4** questions from this Part. **Each** question carries **4** marks.

17. Let ~ be a relation on Z, the set of all integers, defined by $x \sim y$ if $x - y$ is an integer. Is ~ an equivalence relation ? Justify your answer.

18. For $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2 + 1$ and $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = x^2 - 1$, find a formula for $g \circ f$. Hence or otherwise find $g \circ f\,(0)$.

19. Evaluate $\lim\limits_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$.

20. Let $f(x) = \begin{cases} -2 & x \le -1 \\ ax - b & -1 < x < 1 \\ 3 & x \ge 1 \end{cases}$.

For what value of a and b is f continuous at every x ?

21. Does the function $f(x, y) = \frac{x-y}{x+y}$ have a limit as $(x, y) \to (0, 0)$? Justify your answer.

22. Let $u = \sin^{-1} \frac{x^2 + y^2}{x + y}$. Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

23. Find using method of false position, a positive root of the equation $x - e^{-x} = 0$ correct to two decimal places.

PART – D

Answer **any 2** questions from this Part. **Each** question carries **6** marks.

24. a) Let a function f be defined by $f(x) = \frac{3x+2}{x-1}$. Find a formula for f^{-1}.

b) Prove that $\log_b AB = \log_b A + \log_b B$.

25. If $y = \sin^{-1}x$, prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2 y_n = 0$. Further, find $(y_n)0$.

26. Find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at $\left(\frac{1}{2}, 1\right)$ where $w = xy + yz + xz$, $x = u + v$, $y = u - v$ and $z = uv$.

27. Derive the Newton's method for finding 1/N, where N > 0. Hence, find 1/17 correct to four decimal places, using the initial approximation $x_0 = 0.05$.

K21U 6557

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CBCSS – Supplementary)
Examination, November 2021
(2015 – 2016 Admissions)
CORE COURSE IN MATHEMATICS
1B01MAT : Differential Calculus

Time : 3 Hours   Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Find the partial derivative with respect to x of sin (2x + 8y).
2. Find the polar coordinate of $\left(-1, \sqrt{3}\right)$.
3. What are the critical points of f when $f' = x(x-1)$?
4. State Cauchy's mean value theorem.

SECTION – B

Answer **any 8** questions from among the **5** to **14**. These questions carry **2** marks **each**.

5. If $\lim\limits_{x \to 4} \dfrac{f(x) - 5}{x - 2} = 1$, find $\lim\limits_{x \to 4} f(x)$.
6. Show that $\dfrac{df^{-1}}{dx} = \dfrac{1}{\frac{df}{dx}}$ for f = 2x + 3.
7. Translate the equation $\rho = 9 \csc \phi$ into cylindrical and rectangular coordinate systems.
8. Find the rectangular and cylindrical coordinates corresponding to $\left(\sqrt{2}, \pi, \dfrac{\pi}{2}\right)$.

9. Find the rectangular coordinates of the center of the sphere $\rho = 2 \sin \phi (\cos \theta - 2 \sin \theta)$.

10. Evaluate $\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x}$.

11. Find domain and range of the function $w = \frac{1}{\sqrt{16 - x^2 - y^2}}$.

12. Show that $f(x, y, z) = \left(x^2 + y^2 + z^2\right)^{\frac{1}{2}}$ satisfies the Laplace equation.

13. Find $\frac{\partial w}{\partial r}$ if $w = x^2 + y^2$, $x = r + e^s$, $y = \log s$.

14. Find linearisation of $f(x, y, z) = x^2 - xy + 3 \sin z$ at $(2, 1, 0)$.

SECTION – C

Answer **any 4** questions from among the **15** to **20**. These questions carry **4** marks **each**.

15. Evaluate $\int_{\frac{2}{\sqrt{3}}}^{2} \frac{\cos(\sec^{-1} x)}{x\sqrt{x^2 - 1}} dx$.

16. If $y = \cos(m \sin^{-1} x)$, then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$.

17. Find the equation of the sphere which is tangential to the plane $2x + 2y - 2z = 11$ at $(2, 2, 1)$ and passes through the point $(1, 0, -1)$.

18. Find the Maclaurin series of the function $f(\theta) = \sin \theta$.

19. A rectangular sheet of length 6 mtrs. and width 2 mtrs. is given. Four equal squares are removed from the corners. The sides of this sheets are now turned up to form an open rectangular box. Find approximately, the height of the box, such that the volume of the box is maximum.

20. Show that $f(x, y) = \begin{cases} \frac{2x^2y}{x^4 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$ has no limit as (x, y) approaches to $(0, 0)$.

SECTION – D

Answer **any 2** questions from among the **21** to **24**. These questions carry **6** marks **each**.

21. Using definition of limit prove that $\lim_{x \to 2} f(x) = 4$ if $f(x) = \begin{cases} x^2 & x \neq 2 \\ 1 & x = 2 \end{cases}$.

22. Graph the function $y = x^3 (8 - x)$.

23. Prove that the radius of curvature at any point of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ is three times the length of the perpendicular from the origin to the tangent at that point.

24. If $u = \log \frac{x^2 + y^2}{x + y}$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$.